Wk 3 -th

$A; B =$ constants!

Partial Fractions : Idea: $\frac{1}{x^2+x-6} = \frac{1}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2}$

Ex #1 (distinct roots (factors))

$$\int \frac{1}{x^2-7x+12}\,dx \overset{\text{factor}}{=} \int \frac{1}{(x-3)(x-4)}\,dx$$

$$= \int \frac{-1}{x-3} + \frac{1}{x-4}\,dx$$

$$= -\ln|x-3| + \ln|x-4| + C$$

$\frac{1}{4} + \frac{1}{3} = \frac{3}{3}\cdot\frac{1}{4} + \frac{4}{4}\cdot\frac{1}{3}$

$= \frac{3}{12} + \frac{4}{12}$

$= \frac{7}{12}$

① $\frac{1}{(x-3)(x-4)} = \frac{A}{x-3} + \frac{B}{x-4}$

② clear denominators multi both sides by $(x-3)(x-4)$

$$1 = A(x-4) + B(x-3)$$

③ goal: find A & B [this is true for all x]

sub: $x=4 \Rightarrow 1 = A(4-4) + B(4-3) = A\cdot 0 + B\cdot 1 = B$

$x=3 \Rightarrow 1 = A(3-4) + B(0) = A(-1)$

$\Rightarrow A = -1$

Ex 2 Repeated roots (factor)

$$\int \frac{1}{x(x-1)^2}\,dx$$

(Note: this may appear as
$$\int \frac{1}{x^3-2x^2+x}\,dx \Big)$$

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$$\int \frac{1}{x} + \frac{-1}{x-1} + \frac{1}{(x-1)^2}\,dx$$

$$\ln|x| - \ln|x-1| + \int \frac{1}{(x-1)^2}\,dx \quad (A)$$

$u = x-1$

$du = dx$

$= \int \frac{1}{u^2}\,du$

$= \int u^{-2}\,du$

$= \frac{u^{-1}}{-1} + c = \frac{-1}{(x-1)}$

$$\boxed{\ln|x| - \ln|x-1| - \frac{1}{x-1} + C}$$

① recognize a factor in denom is repeated

② get a partial fraction for all "intermediate" factors

$$\frac{1}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

Ex

$$\frac{1}{x(x-1)^4} = \frac{A}{x} + \underbrace{\frac{B}{x-1} + \frac{C}{(x-1)^2} + \frac{D}{(x-1)^3}}_{\text{intermediate}} + \frac{E}{(x-4)^4}$$

③ clear denom:

$$1 = A(x-1)^2 + Bx(x-1) + Cx$$

④ sub: $x = 1 \Rightarrow 1 = A\cdot 0 + B\cdot 0 + C(1) \Rightarrow C = 1$

⑤ Both sides are poly's:

$$C_1x^2 + C_2x + C_3$$

constants

(A) LHS: $0x^2 + 0x + 1 = 1$

RHS: Expand. (but use what you've learned about C)

$$A(x^2-2x+1) + B(x^2-x) + x$$ used $C=1$

$$Ax^2 - 2Ax + A + Bx^2 - Bx + x$$

$-2(1) - (-1) + 1 = 0$ ☺

$$= (A+B)x^2 + (-2A - B + 1)x + A$$

equating coeffs

$\Leftarrow$

$A = 1$

$A + B = 0$

$B = -1$

Ex 3 Irreducible Factor (can't factor degree n) $\Rightarrow$ Irreducible

discriminant : $b^2 - 4ac < 0$

$$\int \frac{1}{x(x^2+1)}\,dx$$

$$= \int \frac{A}{x} + \frac{\overbrace{Bx + C}^{\text{degree } 1}}{x^2+1}\,dx \Rightarrow$$

clear denominators

$$1 = A(x^2+1) + (Bx+C)x$$

$$= Ax^2 + A + Bx^2 + Cx$$

$$= (A+B)x^2 + Cx + A$$

$$\Rightarrow \quad A = 1$$
$$C = 0$$
$$B = -1$$

equating coeffs

same techniques follow

$$= \int \frac{1}{x} + \frac{-x}{x^2+1}\,dx$$

$$\ln|x| - \frac{1}{2}\ln|x^2+1| + C$$