wk 5 - FRI

ex $\left\{ \frac{n}{3} \cdot \sin\left(\frac{8}{n}\right) \right\}$ $\xrightarrow{\text{think}}$ $\frac{\infty}{3} \cdot \sin\left(\frac{8}{\infty}\right) \approx \infty \cdot \sin(0) = \infty \cdot 0$

indeterminate form

Find the limit: $n \to \infty$

$\frac{8}{u} = n$

Set $\frac{8}{n} = u$. As $n \to \infty$, $u \to 0$

So $\frac{n}{3} = \frac{\left(\frac{8}{u}\right)}{3} = \frac{8}{3u}$

sequence becomes $\frac{8}{3u} \cdot \sin(u) = \frac{8}{3} \cdot \frac{\sin(u)}{u}$

$\lim_{u \to 0} \frac{8}{3} \frac{\sin(u)}{u} = \frac{8}{3} \cdot \frac{\overbrace{\sin(0)}^{=0}}{0} = \frac{8}{3} \cdot \frac{0}{0}$ L'H applies $\overset{L'H}{=} \lim_{u \to 0} \frac{8}{3} \cdot \frac{\cos(u)}{1} = \frac{8}{3}$

L.C.T. Limit Comparison Test

For positive sequences only, $\{a_n\}$, $\{b_n\}$, Form : $\lim_{n\to\infty} \frac{a_n}{b_n} = L$

$\sum a_n$

$\sum b_n$

3 cases:

① $L > 0$: $L = \frac{\lim a_n}{\lim b_n} \Rightarrow L \cdot \lim b_n = \lim a_n$

finite positive

The two sequence end up basically the same, i.e., only a factor of L apart

$\sum a_n$ converges (diverges) $\iff$ $\sum b_n$ converge (diverge)

② $L = \infty$: $L = \frac{\lim a_n}{\lim b_n}$ If $\sum a_n$ converges then $\sum b_n$ converges

the a sequence is growing much faster than the b

③ $L = 0$: $L = \frac{\lim a_n}{\lim b_n}$ If $\sum b_n$ converges then $\sum a_n$ converges

the b sequence is growing to infinity much faster than the a

Recall: Series

Converges	Diverges
$\sum_{n=0}^{\infty} \frac{1}{2^n}$ geometric $\lvert r \rvert < 1$ $= \frac{c}{1-r}$	$\sum_{n=1}^{\infty} \frac{1}{n}$ Harmonic
p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ $p > 1$	p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ $p \leq 1$

$$\frac{n^4}{n^5+1} \to \frac{1}{\partial on} \to 0$$

$$\frac{n}{e^n - 1} \to 0$$

ex $\sum \frac{n^2}{n^4 - n + 1}$

converge or diverge

1. choose Test: L.C.T.
2. choose a comparable: Focus on 1^{st} Terms

$$\frac{n^2}{n^4} \underset{\text{algebra}}{=} \frac{1}{n^2}$$

3. $$\lim_{n\to\infty} \frac{\left(\frac{n^2}{n^4-n+1}\right)}{\left(\frac{1}{n^2}\right)} = \lim_{n\to\infty} \frac{n^2}{n^4-n+1} \cdot \frac{n^2}{1} = \lim_{n\to\infty} \frac{n^4}{n^4-n+1}$$

$$= \frac{\infty^4}{\infty^4 + \infty + 1} = \frac{\infty}{\infty}$$

L'H $\frac{0}{0}, \frac{\infty}{\infty}$

$= 1$

4. Interpret step 3

Since $\sum \frac{1}{n^2}$ converges

the 1 ⇒ "same" series

⇒ Convergent

ex $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2+5}}$

1. choose test: L.C.T
2. comparable: $\frac{1}{\sqrt{n^2}} = \frac{1}{n}$

3. $$\lim_{n\to\infty} \frac{\left(\frac{1}{\sqrt{n^2+5}}\right)}{\left(\frac{1}{n}\right)} = \lim_{n\to\infty} \frac{1}{\sqrt{n^2+5}} \cdot \frac{n}{1} = \lim \frac{\sqrt{n^2}}{\sqrt{n^2+5}} = \lim_{n\to\infty} \sqrt{\frac{n^2}{n^2+5}} =$$

$n = \sqrt{n^2}$

$$= \sqrt{\underbrace{\lim_{n\to\infty} \frac{n^2}{n^2+5}}_{=1}} = 1$$

Since $\sum \frac{1}{n}$ diverges this means our series has same behavior

⇒ diverges