4. $\int \tan^3\theta \sec^3\theta \, d\theta = \int \tan^2\theta \sec^2\theta \underbrace{\sec\theta\tan\theta \, d\theta}_{d(\sec\theta)}$

odd tan!

$= \int (\sec^2\theta - 1)(\sec^2\theta) \sec\theta\tan\theta \, d\theta$

$u = \sec\theta$
$du = \sec\theta\tan\theta \, d\theta$

$= \int (u^2 - 1)\cdot(u^2) \, du$

$= \int u^4 - u^2 \, du = \frac{u^5}{5} - \frac{u^3}{3} + C$

$= \frac{\sec^5\theta}{5} - \frac{\sec^3\theta}{3} + C$

5. $\int x^4 \sec^2(x^5) \, dx = \int \cancel{x^4} \cdot \sec^2(u) \frac{1}{5\cancel{x^4}} du = \frac{1}{5}\int \sec^2(u) \, du = \frac{1}{5}\tan(u) + C$

$= \frac{1}{5}\tan(x^5) + C$

u-su

see deriv relation

$u = x^5$

$\frac{du}{dx} = 5x^4$

$du = 5x^4 \, dx$

$\frac{1}{5x^4} du = dx$

6. (you need to do at least three of the following; additional ones may be done for extra credit.)

(6.1) $\int 2x\tan^{-1}x\,dx$

(6.2) $\int 2x\sec^{-1}x\,dx$

(6.3) $\int e^{2x}\sin 3x\,dx$

$\int \sec\theta\,d\theta = \int \sec\theta \cdot \frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta}\,d\theta = \int \frac{\sec^2\theta + \sec\theta\tan\theta}{\sec\theta + \tan\theta}\,d\theta = \ln|\sec\theta + \tan\theta|$

$1-\tan^2\theta$

''

(6.4) $\int \sec^3\theta\,d\theta = \int \sec^2\theta\sec\theta = \int \sec\theta\,d\theta - \int \sec\theta\tan^2\theta\,d\theta$

$u = \sec\theta \quad dv = \sec^2\theta$

$du = \sec\theta\tan\theta \quad v = \tan\theta$

$= \sec\theta\tan\theta - \int \sec\theta\tan^2\theta = \sec\theta\tan\theta - \int \sec^3\theta\,d\theta - \sec\theta\,d\theta = \sec\theta\tan\theta - \int\sec^3\theta\,d\theta + \int\sec\theta\,d\theta$

$(\sec^2\theta - 1)$

add $\int \sec^3\theta$

(6.5) $\int \frac{x^2}{\sqrt{16-x^2}}\,dx$

$\int \sec^3\theta\,d\theta = \frac{1}{2}\left[\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|\right]$

(6.6) $\int \frac{x^2+4x+6}{x(x^2+2x+1)}\,dx$

10.1/10.2/10.3 MA143 Mon Wk 5

Main Question: Do they converge or diverge (not convergent)

Sequences (list)

converge: as you move farther out the sequence you get arbitrarily close to some finite #

diverge: eg, diverge to ∞ $\{n, n+1, n+2, n+3, \ldots\}$

eg diverge $\{1,0,1,0,1,0,\ldots\}$

bounded: Every term is $\leq$ some finite #, i.e., $a_n \leq M$

increasing: $a_n < a_{n+1}$ for all n

decreasing: $a_n > a_{n+1}$

montone: either increasing, or decreasing (or constant)

Series (sum)

converge: sequence of its partial sums converge

all numerators being equal …

bigger denominator means smaller fraction

ex: $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ $\frac{1}{n} \leq 1 \;\forall n$

bounded & monotone

$\frac{1}{n} > \frac{1}{n+1}$ true b/c $n+1 > n$ true

converges to 0

ex: $\left\{\left(\frac{-1}{n}\right)^n\right\}_{n=1}^{\infty}$ bounded: $\leq \frac{1}{2}$

not monotone

$-1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, \ldots$

converges to 0

ex: $\{8,6,7,5,3,0,9,1,2,3,5,1,2,1,2,1,2,\ldots\}$

- not monotone
- bounded ≤ 9.

diverges

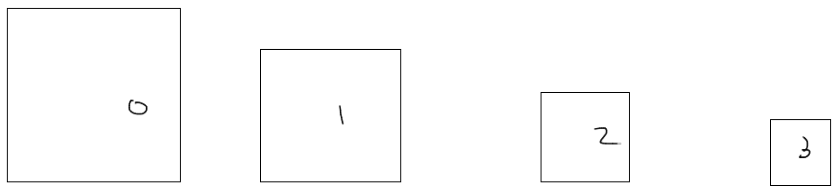
Bounded + Monotone ⇒ Convergence Theorem

If sequence is bounded and monotone then it converges.

Geometric Sequences

- adjacent terms have a common ratio

$$A_n = \left\{5\left(\tfrac{1}{2}\right)^n\right\}_{n=0}^{\infty}$$



$A=5$ $\quad$ $A=\frac{1}{2}(5)$ $\quad$ $A=\frac{1}{4}(5)$ $\quad$ $A=\frac{1}{8}5$ $\quad \ldots$

common ratio $\quad \dfrac{5\left(\frac{1}{2}\right)^3}{5\left(\frac{1}{2}\right)^2} = \frac{1}{2} \quad \Bigg| \quad \dfrac{5\left(\frac{1}{2}\right)^2}{5\left(\frac{1}{2}\right)^1} = \frac{1}{2}$

Geometric Sequence thm:

If the ratio is <1 then it converges.