MA163 Wk 5 Th

Integral Test & Comparison Test:

So far we know:

Series: $\sum_{n=0}^{\infty} a_n = \lim_{N\to 0} \sum_{n=0}^{N} a_n$

Geometric Series: $\sum_{n=0}^{\infty} cr^n \underset{|r|<1}{\overset{\text{when}}{=\!=}} \frac{c}{1-r}$ (DNE otherwise)

Divergence Test: If $\underbrace{\lim_{n\to\infty} a_n}_{\text{sequence of "summands"}} \neq 0$ then it diverges.

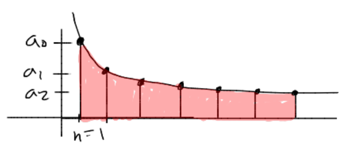
ex $\sum_{n=0}^{\infty} \frac{4n}{5n+1}$ [Since $\lim \frac{4n}{5n+1} \overset{L'H}{=} \frac{4}{5}$ div. test $\Rightarrow$ diverges]

Integral Test

(positive series only)

For $\sum_{n=1}^{\infty} a_n$ $\quad a_n = f(n)$

$\uparrow$ output "#", sample heights of graph



Related to area under curve $= \int_1^{\infty} f(x)\,dx$

If $\underbrace{\int_0^{\infty} f(x)\,dx < \infty}_{\text{converges}}$ then $\sum_{n=0}^{\infty} a_n$ converges

If $\underbrace{\int_0^{\infty} f(x)\,dx < \infty}_{\text{diverges}}$ then $\sum_{n=0}^{\infty} a_n$ diverges

<u>if</u> $f(x)$ is positive continuous & decreasing

$f(x) > f(x-1)$ or $f'(x) < 0$

Ex $\sum_{n=0}^{\infty} \frac{1}{\sqrt{n+3}}$

0. Recognize terms are
   - positive
   - decreasing: $\left(\frac{1}{\sqrt{x+3}}\right)' = -\frac{1}{2}(x+3)^{-1/2} = \frac{\left(-\frac{1}{2}\right)}{\sqrt{x+3}} < 0$
   - cts:

1. $\int^{\infty} \frac{1}{\sqrt{x+3}}\,dx =$ $\quad u = x+3$, $x=1 \Rightarrow u=4$; $du = dx$ $\quad \int_4^{\infty} (u)^{-1/2}\,du = \frac{u^{1/2}}{(1/2)}\Big|_4^{\infty} = 2\sqrt{u}\Big|_4^{\infty} = 2\sqrt{\infty} - 2\sqrt{4} = \infty$ diverges

2. $\Rightarrow$ Series diverges by Integral test

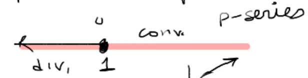
Direct Comparison Test
(positive series only)

given two series

$\sum_{n=0}^{\infty} a_n$ & $\sum_{n=0}^{\infty} b_n$ (bigger) $\qquad 0 \le a_n \le b_n$

Know: possible comparables

$\sum \frac{1}{n}$



Int. Test $\int_1^{\infty} \frac{1}{x} = \ln|x| \Big|_1^{\infty} = \infty$ +f

+ dec.

$\boxed{\text{DIVERGES}}$

If the bigger converges then the smaller does too

If $\sum b_n$ converges the $\sum a_n$ converges

If the smaller series diverges then the larger must too.

ex $\sum_{n=0}^{\infty} \frac{1}{n+3}$ compare to $\frac{1}{n}$

$\frac{1}{n+3} \overset{?}{>} \frac{1}{n}$ b/c then apply comp. test

equivalent to $n > n+3$ ✗ $\therefore$ can not compare to $\sum \frac{1}{n}$

↳ use some other method : eg. integral test

ex $\sum_{n=0}^{\infty} \frac{1}{\sqrt{n}+3}$ try $\frac{1}{\sqrt{n}+3} \overset{?}{>} \frac{1}{n}$, if so $\sum \frac{1}{\sqrt{n}+3}$ diverges by D.C.T.

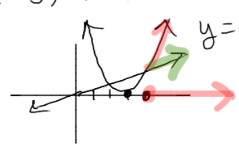
diverges by D.C.T.

(only must be true @ the tail (c, ∞))

$n > \sqrt{n} + 3$

$n - 3 > \sqrt{n}$

$(n-3)^2 > n$ true past point of intersection.



ex $\sum_{n=1}^{\infty} \frac{1}{n+10}$ diverges

Hint: compare to $\sum_{n=1}^{\infty} \frac{1}{2n}$ (diverges)

① factor out $\frac{1}{2}$ ↑ : $\frac{1}{2} \underbrace{\sum_{n=1}^{\infty} \frac{1}{n}}_{\text{diverges!}}$ think: $\frac{1}{2}(\infty) = \infty$

Need $\frac{1}{n+10} \overset{?}{>} \frac{1}{2n}$ True on (n, ∞) but that's enough.

$2n > n+10$

$n > 10$