MA163 W wk 5

geometric series:

$$\sum_{n=0}^{\infty} cr^n = c + cr + cr^2 + \ldots$$

sometimes they converge, eg $\sum_{n=0}^{\infty} 5\left(\frac{1}{2}\right)^n = 10$

sometimes they don't: $\sum_{n=0}^{\infty} \frac{1}{2}(5)^n \rightarrow$ diverges

Key: $|r| < 1 \Rightarrow$ converge (p-series)

$|r| \geq 1 \Rightarrow$ diverge

we actually can compute the # it converges to

$$\sum_{n=0}^{\infty} cr^n = L = c + cr + cr^2 + cr^3 + \ldots$$

Assume $\sum_{n=0}^{\infty} cr^n$ converges

eg, $S_N = \sum_{n=0}^{N} cr^n$ & $\lim_{N\to\infty} S_N = L$

The sequence of partial sums converge

also: $S_{N-1} = \sum_{n=0}^{N-1} cr^n$

clearly $\lim_{N\to\infty} S_{N-1} = L$

(idea $\lim_{n\to\infty} \frac{1}{n} = 0$

$\lim_{n\to\infty} \frac{1}{n-1} = 0$

we actually can compute the # it converges to -

$$\sum_{n=0}^{\infty} cr^n = L = c + cr + cr^2 + cr^3 + \ldots$$

$$- \quad rL = cr + cr^2 + cr^3 + cr^4 + \ldots$$

mult. by r

$$L - rL = c$$

$$L(1-r) = c$$

$$\boxed{L = \frac{c}{1-r}}$$

formula what value (number)
a geometric series converges to

… assuming it converges at all

Ex. $\sum_{n=0}^{\infty} 2\left(\frac{1}{2}\right)^n$

① observe $r = \frac{1}{2} < 1 \Rightarrow$ converges!

② Formula $L = \frac{2}{1-\left(\frac{1}{2}\right)} = 4$

If you just jump to applying
the formula this can happen …

Ex. $\sum_{n=0}^{\infty} \underbrace{\frac{1}{2}}_{c} \cdot \underbrace{2}_{r}^{n} = \frac{\frac{1}{2}}{1-2} = \frac{\left(\frac{1}{2}\right)}{-1} = -\frac{1}{2}$

Ex $\sum 5\left(-\frac{3}{4}\right)^n = \frac{5}{1-\left(-\frac{3}{4}\right)} = \frac{5}{\left(\frac{7}{4}\right)} = \frac{20}{7} \approx 2.9$

Consider the series.

$$\frac{8}{3}+\frac{8}{3^2}+\frac{8}{3^3}+\frac{8}{3^4}+\cdots$$

$$\frac{\left(\frac{8}{3^2}\right)}{\frac{8}{3}} = \frac{1}{3} = r$$

This can be written as a geometric series in the form $\sum_{n=0}^{\infty} cr^n = c + cr^1 + cr^2 + \ldots$

(Give an exact answer. Use symbolic notation and fractions

Set $m = n-2$ so $n = m+2$
then when $n=2$, $m=0$

$$\boxed{\frac{A^n}{B^n} = \left(\frac{A}{B}\right)^n}$$

$$\boxed{(A)^{m+n} = A^m \cdot A^n}$$

$$\sum_{n=2}^{\infty} \frac{7\cdot(\overset{A}{-3})^n}{\underset{B}{4^n}} = \sum_{n=2}^{\infty} 7\left(\frac{-3}{4}\right)^n = \sum_{m=0}^{\infty} 7\left(-\frac{3}{4}\right)^{m+2}$$

$$= \sum_{m=0}^{\infty} \underbrace{7\left(-\frac{3}{4}\right)^m\cdot\left(-\frac{3}{4}\right)^2}_{c} = \sum_{m=0}^{\infty} \frac{63}{16}\left(\frac{-3}{4}\right)^m = \frac{\frac{63}{16}}{\underbrace{1+3/4}_{(7/4)}} = \frac{63}{16}\cdot\frac{4}{7} = \frac{9}{4}$$

Divergence Test: (Test for divergence)
i.e., this idea only tells us if something diverges.

Idea: In order for a series to converge, the n_th term has to get very small as n grows

Assume $\sum a_n$ converges, so if $S_N = \sum_{n=0}^{N} a_n = a_0 + a_1 + \dots + a_N$ then $\lim_{N\to\infty} S_N = L$

and $\lim_{N\to\infty} S_{N-1} = L$

$S_{N-1} = a_0 + a_1 + \dots + a_{N-1}$

Look at the sequence $\{a_N\}$

$$\lim_{N\to\infty} \{S_N - S_{N-1}\} = \underbrace{\lim_{N\to\infty} S_N}_{L} - \underbrace{\lim_{N\to\infty} S_{N-1}}_{L} = 0$$

$\lim a_N = 0$

i.e.,

$$\boxed{\text{If } \lim_{n\to\infty} a_n \neq 0 \text{ then } \sum_{n=0}^{\infty} a_n \text{ diverges.}}$$

Ex $\sum_{n=1}^{\infty} \frac{n}{4n+1}$ does it converge or diverge

Apply div. Test: If $\lim_{n\to\infty} \frac{n}{4n+1} \neq 0 \Rightarrow$ diverge

L'Hospital's Rule

$= \frac{1}{4} \neq 0 \Rightarrow$ diverges

Ex $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ div or conv.

① Div. Test: $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = \frac{1}{\infty} = 0$

(~~fails~~)
(inconclusive)

② diverges by p-test