wk 6 - M

warm-up: Does this converge or diverge.

$$\sum_{n=1}^{\infty} \frac{5}{(n+1)(n+2)(n+3)}$$

ignore all but leading terms

① compare to

② L.C.T.

$$\lim_{n\to\infty} \frac{\left(\cancel{\frac{5}{(n+1)(n+2)(n+3)}}\right)}{\left(\cancel{\frac{5}{n\cdot n\cdot n}}\right)} = \lim_{n\to\infty} \frac{1}{(n+1)(n+2)(n+3)} \cdot \frac{n^3}{1} = 1 > 0$$

③ $\Rightarrow$ same behavior!

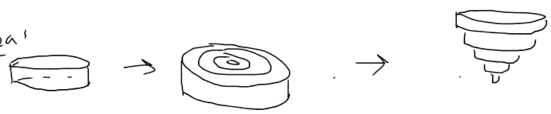
④ $\approx \sum \frac{5}{n^3}$ converges by p-series $p>1$

⑤ Any given series converges by L.C.T.

$$\frac{\frac{1}{B}}{\frac{1}{C}} = \frac{\frac{\cancel{A}}{B}}{\frac{\cancel{A}}{C}} = \frac{\cancel{A}}{B}\cdot\frac{C}{\cancel{A}} = \frac{C}{B}$$

$\boxed{Q}$ To what does this series converge?

idea:



① $$\frac{5}{(n+1)(n+2)(n+3)} = \frac{A}{n+1} + \frac{B}{n+2} + \frac{C}{n+3} = \frac{5/2}{n+1} - \frac{5}{n+2} + \frac{5/2}{n+3}$$

see next page

$$\sum_{n=1}^{\infty} \frac{5}{(n+1)(n+2)(n+3)} = 5\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)(n+3)} = 5\sum \frac{A}{n+1} + \frac{B}{n+2} + \frac{C}{n+3}$$

$A = 1$, $B = -2$, $C = 1$

$$= 5\sum \frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}$$

$$\underbrace{\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}}_{n=1} + \underbrace{\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}}_{n=2} + \underbrace{\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}}_{n=3} \ldots$$

$S_1: \frac{1}{2} - \frac{2}{3} + \frac{1}{4}$

$S_2: \frac{1}{2} - \frac{2}{3} + \frac{1}{4} + \frac{1}{3} - \frac{2}{4} + \frac{1}{5}$

$S_3: \frac{1}{2} - \frac{2}{3} + \frac{1}{4} + \frac{1}{3} - \frac{2}{4} + \frac{1}{5} + \frac{1}{4} - \frac{2}{5} + \frac{1}{6}$

$S_4: \frac{1}{2} - \frac{2}{3} + \frac{1}{4} + \frac{1}{3} - \frac{2}{4} + \frac{1}{5} + \frac{1}{4} - \frac{2}{5} + \frac{1}{6} + \frac{1}{5} - \frac{2}{6} + \frac{1}{7}$

less than the level
... nothing will cancel
(residue)

write in terms of 4

$\underbrace{\frac{1}{2} - \frac{2}{3}} + \frac{1}{3}$

$\frac{5}{6} - \frac{2}{3} = \frac{1}{6}$

$S_K = \frac{1}{6} - \frac{1}{K+2} + \frac{1}{K+3}$,

Ans: $5\sum \frac{1}{(n+1)(n+2)(n+3)} = 5 \cdot \lim_{K\to\infty} S_K$

$5 \cdot \left(\frac{1}{6} + 0 + 0\right) = \frac{5}{6}$

Some new ideas:

Absolute Convergence: "The sum of the absolute values converge"

ex $\sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n$

$= (-1)^n \quad + - + - + - \ldots$

① does the seq. converge? (alt. series test) $\Rightarrow$ If alt & decreasing $|a_n|$ then converge

$|a_n| = \left(\frac{1}{2}\right)^n = \frac{1}{2^n} \rightarrow$ decreasing

yes conv. by A.S.T.

② does it converge absolutely?

take abs value:

$\sum \left(\frac{1}{2}\right)^n$ converges b/c geometric w/ $r = \frac{1}{2} < 1$

$\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3} \rightarrow$ ratio: $\frac{1}{2} = r$

$\left(\frac{1}{n}\right)^p = \frac{1}{n^p}$

ex $\sum_{n=0}^{\infty} (-2)^n$ ① does it converge? NO

ex $\sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

① converge, yes, A.S.T. (i) alt (ii) decr. $\frac{1}{n} > \frac{1}{n+1}$

② converge absolutely? (NO) .... $\left|\frac{-1^n}{n}\right| = \frac{1}{n} = \sum \frac{1}{n}$ Harmonic divergent