ex

$$\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)^2+\left(\frac{1}{2}-\frac{1}{3}\right)^3+\dots$$

$\frac{1}{2}-\frac{1}{3}$

$\frac{3}{6}-\frac{2}{6}=\frac{1}{6}$

$$\frac{1}{6}+\left(\frac{1}{6}\right)^2+\left(\frac{1}{6}\right)^3+\dots=\sum_{n=1}^{\infty}\left(\frac{1}{6}\right)^n$$

1st term $=\frac{1}{6}=c$

common ratio $=\frac{1}{6}$

- NO AST b/c $\frac{1}{6}>0$

$\left(\frac{1}{3}-\frac{1}{2}\right)=-\frac{1}{6}$

geometric series

$$\sum_{n=0}^{\infty} c\cdot r^n=\frac{c}{1-r}$$

$c=$ 1st term

$r=$ common ratio

- $a_n=\frac{1}{6}$

$$a_n=\frac{1}{2^n}-\frac{1}{3^n}$$

$$(A-B)^n\neq A^n-B^n$$

$$=\boxed{\frac{c}{1-r}}=\frac{\frac{1}{6}}{1-\frac{1}{6}}=\frac{\cancel{\frac{1}{6}}}{\cancel{\frac{5}{6}}}=\frac{1}{5}$$

$$S=\frac{1}{2}-\frac{1}{3}+\frac{1}{2^2}-\frac{1}{3^2}+\frac{1}{2^3}-\frac{1}{3^3}+\dots$$

swap order (this works b/c the series converges)

$$=\underbrace{\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}}_{\text{geometric series}}+\dots\underbrace{-\frac{1}{3}-\frac{1}{3^2}-\frac{1}{3^3}-\dots}_{-\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\dots\right)}$$

$c=\frac{1}{2}$

$r=\frac{1}{2}$

$$-\left(\underbrace{\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\dots}_{\text{geom. series}}\right)$$

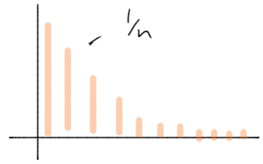
$c=\frac{1}{3}$, $r=\frac{1}{3}$

$$=\frac{\frac{1}{2}}{1-\frac{1}{2}}-\frac{\frac{1}{3}}{1-\frac{1}{3}}=\frac{\frac{1}{2}}{\frac{1}{2}}+\frac{\frac{1}{3}}{\frac{2}{3}}=1-\frac{1}{2}=\frac{1}{2}$$

ex Conditional Convergence –

" converges but only b/c it's alternating and terms cancel.

ex $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges conditionally



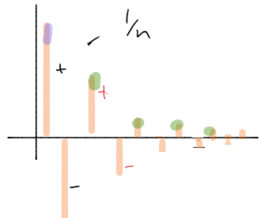
→ sum ∞ height

A.S.T. : alt. series test

For $\sum a_n$

if series is alt. (+, −, + −, ...)

$= \begin{cases} \sum (-1)^n \cdot b_n \\ \text{w/ } b_n \geq 0 \end{cases}$



If the alt series converges $\boxed{\text{AND}}$ it's positive part (absolute value) does too

∴ we say it converges absolutely.

and decreasing positive part, subsequent terms less than previous $\{ b_{n+1} \leq b_n$

⇒ then $\sum a_n$ converges.

ex $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converges absolutely ex

see: factorial? think ratio test

$$\sum_{n=1}^{\infty} \frac{(n!)^3}{(6n)!}$$

① $\lim_{n\to\infty} \dfrac{\dfrac{((n+1)!)^3}{(6(n+1))!}}{\dfrac{(n!)^3}{(6n)!}}$

Ratio Test	
<1	$\Rightarrow$ conv.
>1	$\Rightarrow$ div
$=1$	$=$?

since $0<1 \Rightarrow$ converges

$\approx \dfrac{n^3}{6^6 n^6} \to 0$

$(n+1)n!$ $(n+1)n!$ $(n+1)n!$

$(n+1)!$ $(n+1)!$ $(n+1)!$

② $\lim_{n\to\infty} \dfrac{((n+1)!)^3}{(6(n+1))!} \cdot \dfrac{(6n)!}{(n!)^3} = \lim_{n\to\infty} \dfrac{(n+1)^3}{(6n+6)(6n+5)(6n+4)\cdots(6n+1)} = 0$

$n!\ n!\ n!$

$(6n+6)!$

$= (6n+6)(6n+6-1)(6n+6-2)(6n+3)(6n+2)(6n+1)(6n)!$

Factorials Fun

$\dfrac{5!}{3!} = \dfrac{5\cdot4\cdot3\cdot2\cdot1}{3\cdot2\cdot1} = 20$

$= \dfrac{5\cdot4\cdot3!}{3!}$

$(4n)! = 4n\cdot(4n-1)\cdot(4n-2)\cdots$

increment n

$n \longmapsto (n+1)$

$(4(n+1))! =$

$4(n+1)\cdot(4(n+1)-1)\cdot(4(n+1)-2)\cdots$

$(4n+4)(4n+3)(4n+2)(4n+1)(4n)!$