$$\sum_{n=5}^{\infty} \ln\left(1+\frac{1}{n}\right) = \sum_{n=5}^{\infty} \ln(n+1) - \ln(n)$$

$$\lim_{k\to\infty} S_k$$

kth partial sum

$S_k = ?$

$$\ln\left(1+\frac{1}{n}\right) = \ln\left(\frac{n}{n}+\frac{1}{n}\right) = \ln\left(\frac{n+1}{n}\right) = \ln(n+1) - \ln(n)$$

use this to build telescoping series

use: $\ln(A\cdot B) = \ln(A) + \ln(B)$ $\qquad (e^A)^B = e^{AB}$

$\ln\left(\frac{A}{B}\right) = \ln A - \ln B$ $\qquad e^A\cdot e^B = e^{A+B}$

$S_1 = \ln(2) - \overset{=0}{\ln(1)}$

$S_2 = \ln(2) + \ln(3) - \ln(2)$

$S_3 = \ln(\ln 3) + \ln(4) - \ln(3) \quad \ldots$

$S_k = \ln(k+1)$

10.6 Power Series

"How do calculators work?"

ex

$\boxed{e^x} \longrightarrow e^2 \approx 7.389\ldots$

1st It understands "polynomials" — it knows exponents, +, −

Key $e^x \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$

$\boxed{x} \rightarrow 3x = 3 \cdot x$
$\uparrow$ multi

$\boxed{x} \longrightarrow x^2 \longrightarrow x \cdot x$

$\boxed{5x^2 + 7x} \longrightarrow$ combinations of $\cdot$ $\frac{1}{\xi}$ +.

2nd It knows power series representations for common functions, e^x, $\sin x$, ...

3rd When you type it e^2 ... it approximates $\sum_{k=0}^{15} \frac{2^n}{n!}$

Power Series

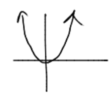
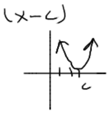
Two conventions: $0^0 = 1$, ..., $(\frac{1}{4})^0 = 1$, $(\frac{1}{2})^0 = 1$, $1^0 = 1$, $2^0 = 1$, $3^0 = 1$

$0! = 1$

Basic Form of Power Series: $\sum_{n=0}^{\infty} a_n (x-c)^n$

often $c = 0$ ...

$\sum_{n=0}^{\infty} a_n x^n$

$y = f(x)$  $y = f(x-c)$ 

the c just shifts the "base" of the power series.

ex $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

since x is a variable some values of x may affect convergence

ex $\sum_{n=0}^{\infty} x^n$ ← what values of x make this converge?

$\boxed{e^x}$ Test for conv: see factorial use ratio test

root test: converge if

$\lim_{n\to\infty} \sqrt[n]{|x^n|} = |x| < 1$

$x \in (-1, 1)$

ex $\sum (\frac{1}{2})^n$ conv, but $\sum (2)^n$ div.

$|x^n \cdot x| = |x^n| \cdot |x|$

$$\lim_{n\to\infty} \frac{\left|\frac{x^{n+1}}{(n+1)!}\right|}{\left|\frac{x^n}{n!}\right|} = \lim_{n\to\infty} \frac{|x^{n+1}|}{(n+1)!} \cdot \frac{n!}{|x^n|} = \lim_{n\to\infty} \frac{|x|}{n+1} = 0$$

$(n+1)! = (n+1)n!$

$\Rightarrow$ converges for all x

By the ratio test: $>1 \Rightarrow$ div

$<1 \Rightarrow$ conv

$=1 \Rightarrow$?

your e^x on your calc, doesn't change accuracy w/ x.