No class tomorrow!

Today: wk 7 Thur

Key Idea: when $\sum_{n=0}^{\infty} a_n (x-c)^n$ converges on $I = (u, v)$ (eg, $(-\infty, \infty)$, $(-1, 1)$, $[-5, 7)$)

manipulations are valid in that interval.

(ex) $f(x) = \sum a_n (x-c)^n$

$$\frac{d}{dx}(f(x)) = \frac{d}{dx}\left(\sum a_n (x-a)^n\right) = \sum a_n \frac{d}{dx}(x-a)^n$$

outside — inside

not dependant on x

i.e., $\frac{d}{dx}(5x^2) = 5\frac{d}{dx}(x^2)$

(ex) geometric series with $x = r$ & $|x|<1$ and $c = 1$.

① write down the series: $1 + x + x^2 + x^3 + x^4 + \ldots =$

② Find the sum, i (formula): $\frac{c}{1-r} = \frac{1}{1-x}$

So $\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \ldots$ $(|x|<1)$

converges on $I = (-1, 1)$

(ex) replace $x = -x^2$, both sides

$$\frac{1}{1-(-x^2)} = 1 + (-x^2) + (-x^2)^2 + (-x^2)^3 + (-x^2)^4 + \ldots$$

$$\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + x^8 + \ldots$$ converges on $(-1,1)$

(we'll see: π = infinite series
use this to write $\pi = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \ldots\right)$
alt. odd recips)

(ex) write power series representation for this & give its interval of convergence.

$f(x) = \dfrac{1}{2+x^2}$

Ratio of Conv. : solve

$\left| -\dfrac{x^2}{2} \right| < 1$

$-1 < \dfrac{x^2}{2} < 1$

$\underbrace{-2 <}_{} \overbrace{x^2}^{} < 2$ ↳ always $> 0 > -2$

$x < \sqrt{2} = \sqrt{2}$

$\Rightarrow |x| \leq \sqrt{2}$

$x \in (-\sqrt{2}, \sqrt{2})$

$x^2 < 2$

$\Rightarrow x < \sqrt{2}$

$-x > -\sqrt{2}$

(idea) make this look like

$g(u) = \dfrac{1}{1-u}$

use $= 1 + u + u^2 + u^3 + \dots$

$\underline{|u| < 1}$

$f(x) = \dfrac{1}{2(1+\frac{1}{2}x^2)} = \dfrac{1}{2(1+\frac{x^2}{2})} = \dfrac{1}{2(1-(-\frac{x^2}{2}))} = \dfrac{1}{2} \cdot \dfrac{1}{1-(-\frac{x^2}{2})}$

$u = -\dfrac{x^2}{2}$ $= \dfrac{1}{2} \cdot \dfrac{1}{1-u} = \dfrac{1}{2}\left(1 + u + u^2 + u^3 + \dots\right) = \dfrac{1}{2}\sum_{n=0}^{\infty} u^n$

$= \boxed{\dfrac{1}{2}\sum_{n=0}^{\infty}\left(-\dfrac{x^2}{2}\right)^n}$

i.e., $\frac{1}{2}\sum r^n$, need $|r| < 1$

Exam Prep:

① give an increasing sequence that converges to π.

Note:

$\{3^n\}$ is increasing ... $3^0, 3^1, 3^2$

$3^2 > 3^1$ ..

$3^x \xrightarrow{d/dx} 3^x \ln 3 > 0 \quad \forall x \Rightarrow$ increasing,

general strategy: start with your target, then add or subtract terms (ultimately vanishing) that make the resulting sequence increase or decrease,

$n=0$

$$\left\{\pi - \frac{1}{3^n}\right\} = \left\{\pi - 1, \pi - \frac{1}{3}, \pi - \frac{1}{3^2}, \pi - \frac{1}{3^3} \ldots\right\}$$

$f(x) = \pi - \frac{1}{3^x} = \pi - 3^{-x}$

bigger denom $\Rightarrow$ smaller fraction

$-a^u \to -a^u du \cdot \ln(a)$

$f'(x) = -3^{-x} \cdot (-1)\ln 3 = 3^{-x} \ln 3 = \frac{1}{3^x} \ln 3 > 0$ ☺

② For a decreasing seq. conv to π ____

$\frac{1}{3^n} > \frac{1}{3^{n+1}} \Leftrightarrow 3^{n+1} > 3^n$ ☺

$$\pi + \frac{1}{3^n} = \left\{\pi + 1, \pi + \frac{1}{3}, \pi + \frac{1}{3^2} \ldots\right.$$

$\pi + \frac{1}{3^n} > \pi + \frac{1}{3^{n-1}}$