Wed. wk 8

Exam 2 : Next Wed (study Guide)

: power series hw posted tonight, (solutions soon)

Friday: MA163 Fall-break: no class

Monday: Review

Today: Power Series : (power of variable x)

$$\sum_{n=0}^{\infty} a_n (x-c)^n$$

variable!

the question is not "does it converge"
(we dont know what x is)
a_n = given

Instead, the question is:
For what x-values does it converge?

ex

$$\sum_{n=0}^{\infty} a_n (x-c)^n$$

1) $a_n = 7$, $x = 1$, $c = 3$

$$\sum_{n=0}^{\infty} 7(1-3)^n = \sum_{n=0}^{\infty} 7(-2)^n$$

diverges b/c geom. series w/ $|r| > 1$

$$= 7(-2)^0 + 7(-2)^1 + 7(-2)^2 + 7(-2)^3 + \dots$$

-2 $\quad \frac{7(-8)}{7(4)} = -2 \Rightarrow$ geometric

$$=$$

$$= \frac{c}{1-r}$$ (only when $|r| < 1$, otherwise it DNE)

$$|r| = 2$$

ex

$a_n = \frac{1}{n^2}$, $x = 4$, $c = 3.5$ — does it conv.?

$$\sum_{n=1}^{\infty} \frac{1}{n^2}(4-3.5)^n = \sum \frac{1}{n^2}\left(\frac{1}{2}\right)^n = \sum_{n=1}^{\infty} \frac{1}{n^2 \cdot 2^n}$$

Ratio

$$\lim_{n\to\infty} \left| \frac{\frac{1}{(n+1)^2 \cdot 2^{n+1}}}{\frac{1}{n^2 \cdot 2^n}} \right| = \lim_{n\to\infty} \left| \frac{1}{(n+1)^2 \, 2^{n+1}} \cdot \frac{n^2 \cdot \cancel{2^n}}{1} \right| = \lim_{n\to\infty} \frac{n^2}{(n+1)^2 \cdot 2} = \frac{\infty}{\infty} \overset{L'H}{=}$$

$2^{n+1} = \cancel{2^n} \cdot 2$

$$= \lim \frac{2n}{2(n+1) \cdot 2} = \frac{\infty}{\infty} \quad L'H$$

$$= \lim \frac{2}{4} = \frac{1}{2} < 1$$

$\Rightarrow$ converges

ex $\sum_{n=0}^{\infty} \frac{(x-2)^n}{(n+2)3^n}$

For what values does the series converge?

For large n

$\frac{n+2}{n+3} \approx 1$

Ratio:

(1) copy OG $\lim_{n\to\infty} \frac{\left|\frac{(x-2)^{n+1}}{((n+1)+2)\cdot 3^{n+1}}\right|}{\frac{(x-2)^n}{(n+2)3^n}} = \lim_{n\to\infty} \left| \frac{(x-2)^{n+1}}{((n+1)+2)\cdot \underbrace{3^{n+1}}_{3^n \cdot 3}} \cdot \frac{(n+2)3^n}{(x-2)^n} \right| = \lim_{n\to\infty} \left| \frac{x-2}{3} \cdot \frac{n+2}{n+3} \right|$

$= \left| \frac{x-2}{3} \right|$

lim vanishes @ same step as does $\frac{n+2}{n+3}$

(2) $\left| \frac{x-2}{3} \right| < 1$

$-1 < \frac{x-2}{3} < 1$

$-3 < x-2 < 3$

$-1 < x < 5$

when $x \in (-1,5)$ converges

(3) Test $x=-1$, $x=5$ individually

plug $x=-1$ into OG

$\sum_{n=0}^{\infty} \frac{(-3)^n}{(n+2)3^n} = \frac{(-1\cdot 3)^n}{} = \frac{(-1)^n \cdot \cancel{3^n}}{(n+2)\cancel{3^n}} = \sum \frac{(-1)^n}{n+2}$

A.S.T. $\Rightarrow$ Converge

$x=5 \Rightarrow \sum \frac{3^n}{(n+2)3^n} = \sum_{n=0}^{\infty} \frac{1}{n+2}$

$\Rightarrow$ diverges!

L.C.T. w/ Harmonic

$\lim \frac{\frac{1}{n+2}}{\frac{1}{n}} = 1 \Rightarrow$ Behaves like Harmonic $\sum 1/n$

$\leftarrow 3 \rightarrow \leftarrow 3 \rightarrow$

-1 2 5

(4) Interval of Convergence

$[-1, 5)$

radius of conv. $\Rightarrow \frac{5-(-1)}{2} = 3$

ex we saw

$$\sum \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots = e^x$$

we'll see this converges for all x (b/c $a_n = \frac{1}{n!}$)

Idea: when a series a converges, you can do 'calculus' on it

$$\frac{d}{dx}(e^x) = \frac{d}{dx}\left(1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots\right)$$

$$\| \qquad\qquad\qquad \|$$

$$e^x \qquad 0 + 1 + \frac{\cancel{2}x}{\cancel{2}} + \frac{\cancel{3}x^2}{\cancel{3}\cdot 2!} + \frac{\cancel{4}x^3}{\cancel{4}\cdot 3!}$$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = e^x$$