Wed. wk 9

Degree 4 approx to

$$f(x) = \frac{\sin(x)}{1-x} = \frac{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots}{1-x} = \left(\frac{1}{1-x}\right) \cdot \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\right)$$

geom

$\sum r^n = \frac{c}{1-r}$

$c = 1$

$r = x$

$\sum_{n=0} x^n = \frac{1}{1-x}$

$$\approx \left(\underbrace{1 + x + x^2 + x^3}_{\deg 4-1} + x^4\right)\left(\underbrace{x - \frac{x^3}{3!}}_{\deg 4-1} + \cancel{\frac{x^5}{5!}}\right)$$ ignore

$\leadsto$ deg 9 approx!

$$\approx x - \frac{x^3}{3!} + x^2 - \frac{x^4}{3!} + x^3 - \frac{x^5}{3!} + x^4 - \frac{x^6}{3!} \approx x + x^2 + \left(1 - \frac{1}{3!}\right)x^3 + \left(1 - \frac{1}{3!}\right)x^4$$

ex

$$\lim_{x \to 0} \frac{\sin 9x - 9x + \frac{243x^3}{2}}{x^5}$$

$$= \lim_{x \to 0} \frac{\cancel{9x} - \frac{(9x)^3}{3!} + \frac{(9x)^5}{5!} \cancel{- 9x} + \frac{243x^3}{2}}{x^5}$$

$$= \lim_{x \to 0} \frac{\left(\frac{9^5}{5!}\right) \cancel{x^5}}{\cancel{x^5}} = \frac{9^5}{5!}$$

3

9·9·9

$$\frac{9^3}{3!} = \frac{(3^2)^3}{3 \cdot 2} = \frac{3 \cdot 81}{2} = \frac{243}{2}$$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$

MacLaurin Series —

choose some depth ($n = 5$) b/c of denom.

$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!}$

so sub $x = 9x$

$\sin 9x \approx 9x - \frac{(9x)^3}{3!} + \frac{(9x)^5}{5!}$

ex

#5, 10.8-3

Expand $\frac{1}{8+7x}$ into $\boxed{\sum_{n=0}^{\infty} a_n x^n \quad \text{w/ } c=0}$ Find $a_n x^n$

$\sum \frac{f^{(n)}(0)}{n!} x^n$

this representation of function into power series is unique.

" Maclaurin Series has $c=0$ (center)

this must be what we're looking for

Find Macl. Series

$f(x) = \frac{1}{8+7x} = (8+7x)^{-1}$ $\quad f(0) =$

$f'(x) = -7(8+7x)^{-2}$

$f''(x) = 2\cdot 7^{2}(8+7x)^{-3}$

$f'''(x) = -3\cdot 2\cdot 7^{3}(8+7x)^{-4}$

$4\cdot 3\cdot 2$
$4!$

$f^{(n)}(x) = (-1)^n \cdot n!\, 7^{n}(8+7x)^{-(n+1)}$ $\quad f^{(n)}(0) = (-1)^n n!\, 7^n (8+7\cdot 0)^{-(n+1)} = \frac{(-1)^n n!\, 7^n}{8^{n+1}}$ ($8+7\cdot 0 = 8$)

$= (-1)^n n! \left(\frac{1}{8}\right)\left(\frac{7}{8}\right)^n$

$\frac{7^n}{8^{n+1}} = \frac{7^n}{8^n 8} = \left(\frac{7}{8}\right)^n \cdot \frac{1}{8}$

$\sum \frac{f^{(n)}(0)}{n!} x^n = \sum_{n=0}^{\infty} \frac{(-1)^n 7^n}{8^{(n+1)}} x^n$

To find [Interval of convergence] Hit w/ Ratio Test & solve the inequality.

$\sum \frac{f^{(n)}(0)}{n!} x^n = \underbrace{\sum_{n=0}^{\infty} \frac{(-1)^n 7^n}{8^{(n+1)}}}\, x^n$

Ratio Test:

$7^{n+1} = 7^n \cdot 7$

$\lim_{n\to\infty} \left| \frac{\frac{7^{n+1} x^{n+1}}{8^{n+2}}}{\frac{7^n \cdot x^n}{8^{n+1}}} \right| = \lim \left| \frac{7^{n+1} x^{n+1}}{8^{n+2}} \cdot \frac{8^{n+1}}{7^n x^n} \right| = \lim_{n\to\infty} \left|\frac{7x}{8}\right| = \underbrace{\left|\frac{7x}{8}\right| < 1}$ sandwich

$8^{n+1} = 8^n \cdot 8$, $\quad 8^{n+2} = 8^n \cdot 8^2$

$-1 < \frac{7x}{8} < 1$

$-8 < 7x < 8$

$-\frac{8}{7} < x < \frac{8}{7}$

$\Rightarrow$ when $x \in \left(-\frac{8}{7}, \frac{8}{7}\right)$ it converges.

when $\left|\frac{7x}{8}\right| = 1 \Rightarrow$ look @ both endpoints

$\boxed{x = -\frac{8}{7}} \Rightarrow \sum (-1)^n \frac{7^n}{8^{n+1}} \left(-\frac{8}{7}\right)^n = -\sum_{n=0}^{\infty} (-1)^n \frac{1}{8} = -\frac{1}{8} + \frac{1}{8} - \frac{1}{8} \cdots \Rightarrow$ div div. test

$x = \frac{8}{7} \Rightarrow \sum (-1)^n \frac{1}{8}$ div.

I.O.C. $\left(-\frac{8}{7}, \frac{8}{7}\right)$