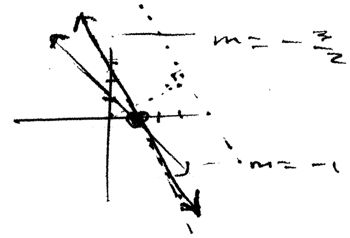
WHY G·J Elimination WORKS

VECTOR: ARROW FROM (0,0) to (a,b).



START WITH THESE EQN'S.

$x + y = 1$

$3x + 2y = 5.$

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 5 \end{bmatrix} \quad \text{(I)}$$

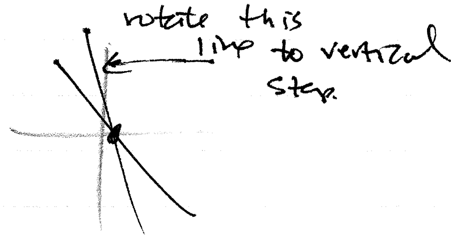
EQN. $\rightarrow$
SOL'S ARE x_0, y_0 s.t.
$x_0 + y_0 = 1$
and
$3x_0 + 2y_0 = 5$
pts "lying on line"
//
"pts satisfying eqn"

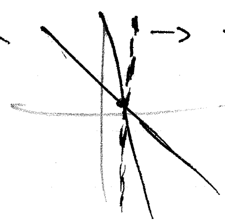
The NORMAL ($\perp$) vector is coef's of LHS.

Goal is to solve system in such a way that scales.

Idea:

start





$\rightarrow$ this line has eq.

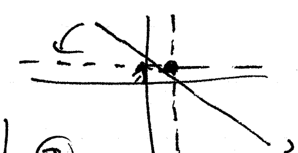
~~$x + 0y = 1.5$~~ ?

$x + 0y = .9$

$\Rightarrow \perp$ to $\leftrightarrow$

(, The rotate this on

Effectively, you got two new eq's like this



$\rightarrow$ this line.

$0x + y = 3$, say 2

$1x + 0y = 2.$

$0x + 1y = 1$

$$\longleftrightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix} \quad \text{(II)}$$

$\Rightarrow$ So you've moved from (I) to (II) through rotations, using fact: if

$ax + by = c$

$dx + ey = f$ $\quad \forall\ a,b,c,d,e,f \in \mathbb{R}.$
—fixed—.

are eq.s. & x_0 & y_0 satisfy both.

This suggests why col'm swapping is a NO-NO.

$ax_0 + by_0 = c$

$dx_0 + ey_0 = f.$ $\quad$ let's add

$\Rightarrow (a+d)x_0 + (b+e)x_0 = c+f$

New eq'n.
New line,
& x_0, y_0 satisfies eq.
$\Rightarrow x_0 y_0$ lies in line.

also

$max_0 + mby_0 = mc$

$ndx_0 + ney_0 = nf$

$$= \begin{matrix} m\,(ax_0 + by_0 = c) \\ n\,(dx_0 + ey_0 = f) \end{matrix}$$

$(ma + nd)x_0 + (mb + ne)y_0 = mc + nf.$

New eq.
New line

Math 211 - Day 1 - 1.1, 1.2 & 1.3 NEED: Color Chalk

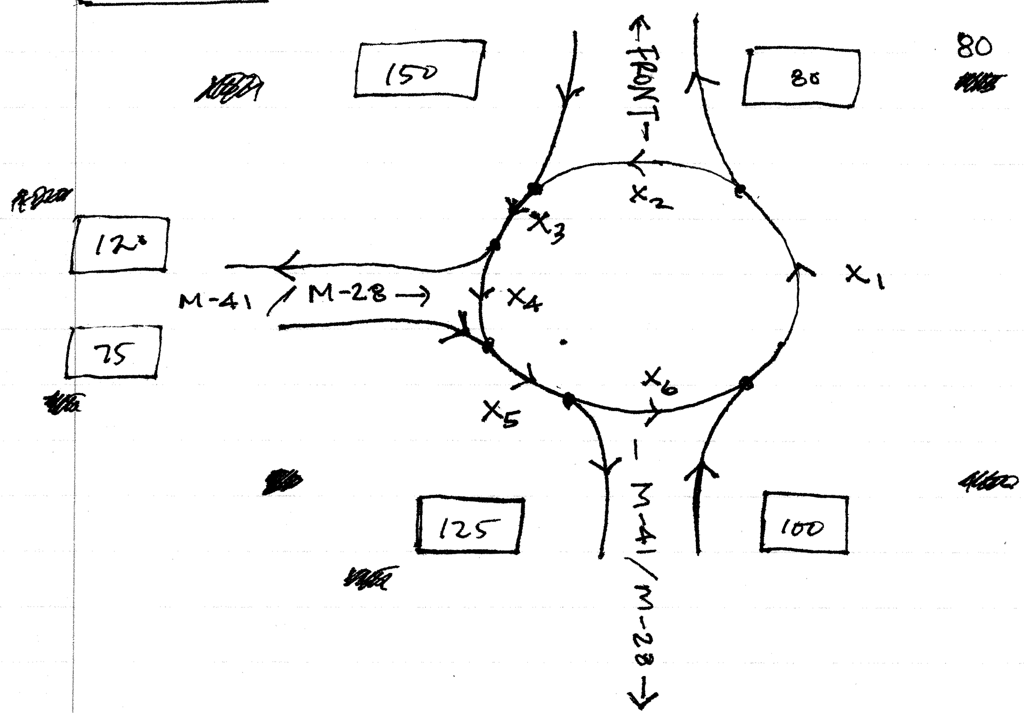
Lastime : Intro & vector spaces.

this day : MQT round-about linear problem, G-J elimination 2 ways geometry & algorithm.

Homework : 1.1.12b, 1.2.6abc, 1.2.12

Next day : Work towards Schwartz's inequality & Δ-inequality.

$\boxed{\text{EXAMPLE}}$ MQT Traffic Round-about



IN TRAFFIC
325 vph

Out Traffic
325 vph

Guiding Principles :
- Traffic in $\Rightarrow$ Traffic out
- Intersections determine equations.

$100 + x_6 = x_1$		$x_1 + 0x_2 + 0x_3 + 0x_4 + 0x_5 - x_6 = 100$
$x_1 - x_2 = 80$		$x_1 - x_2 + 0x_3 + 0x_4 + 0x_5 + 0x_6 = 80$
$x_2 + 150 = x_3$	rewrite	$0x_1 + x_2 - x_3 + 0x_4 + 0x_5 + 0x_6 = -150$
$x_3 - 120 = x_4$		$0x_1 + 0x_2 + x_3 - x_4 + 0x_5 + 0x_6 = 120$
$75 + x_4 = x_5$		$0x_1 + 0x_2 + 0x_3 + x_4 - x_5 + 0x_6 = -75$
$x_5 - 125 = x_6$		$0x_1 + 0x_2 + 0x_3 + 0x_4 + x_5 - x_6 = 125$

The associated matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 100 \\ 1 & -1 & 0 & 0 & 0 & 0 & 80 \\ 0 & 1 & -1 & 0 & 0 & 0 & -150 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 0 & -75 \\ 0 & 0 & 0 & 0 & 1 & -1 & 125 \end{bmatrix}$$

$\downarrow$ by row operations

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 100 \\ 0 & 1 & 0 & 0 & 0 & -1 & 20 \\ 0 & 0 & 1 & 0 & 0 & -1 & \cancel{}\,170 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 125 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

x_6 is free, so $x_6 = t$ for any (positive) value of $t \in \mathbb{R}$.

Rewriting as equations:

$$\begin{aligned} x_5 &= 125 + t \\ x_4 &= 50 + t \\ x_3 &= 170 + t \\ x_2 &= 20 + t \\ x_1 &= 100 + t \end{aligned}$$

Since $t \in \mathbb{R}$, it's free but we should assume $x_i \geq 0$

Ex. $$\begin{aligned} ax + by &= 0 \\ cx + dy &= 0 \end{aligned} \quad (*) \quad (a,b,c,d \text{ fixed reals})$$

Show that if $x = x_0$, $y = y_0$ is a sol'n, then $x = kx_0$, $y = ky_0$ is too, for any const. k.

Before we prove it in general, let's make sure it works in simplicity.

Special Case: $$\begin{aligned} \cancel{1}x + \cancel{1}y &= 0 \\ 3x + 4y &= 0 \end{aligned} \iff \begin{aligned} x + 2y &= 0 \\ 3x + 4y &= 0. \end{aligned}$$

Someone find a sol'n: HINT: RHS = 0, so $x = 0$, $y = 0$ is always a sol'n.

So for say $k = 5$, $x = 5 \cdot 0 = 0$, $y = 5 \cdot 0 = 0$ is also a sol'n

Let's generalize:

PROOF:

Assume $x = x_0$, $y = y_0$ is a sol'n to $(*)$

This says then that $$\begin{aligned} ax_0 + by_0 &= 0 \\ cx_0 + dy_0 &= 0. \end{aligned} \quad (\#)$$

In order for (kx_0, ky_0) to be a sol'n we must show:

$$\begin{aligned} a(kx_0) + b(ky_0) &= 0 \\ c(kx_0) + d(ky_0) &= 0 \end{aligned}$$ using $(\#)$

Multiply $(\#)$ by k: $$\begin{aligned} k(ax_0 + by_0 = 0) &= a(kx_0) + b(ky_0) = 0 \\ k(cx_0 + dy_0 = 0) &= c(kx_0) + d(ky_0) = 0 \end{aligned}$$ by distributive & commutative properties.

Next, show if $x = x_0, y = y_0$ & $x = x_1, y = y_1$ are BOTH sol's ...

then $x = x_0 + x_1, y = y_0 + y_1$ is yet another sol'n.

Proof: (Must use assumptions to find common ground on which you & reader can agree).

Assume $ax_0 + by_0 = 0$ & $ax_1 + by_1 = 0$
$cx_0 + dy_0 = 0$ & $dx_1 + dy_1 = 0$

(★) (This is precisely what it means to be sol's)

We're trying to show something about $x_0 + x_1$, $y_0 + y_1$ so it makes sense to add eq's...

$$ax_0 + by_0 + ax_1 + by_1 = 0 + 0 = 0$$
$$cx_0 + dy_0 + cx_1 + dy_1 = 0 + 0 = 0$$

$$a(x_0 + x_1) + b(\cancel{x_0}\, y_0 + y_1) = 0$$
$$c(x_0 + x_1) + d(y_0 + y_1) = 0$$

by (★)

we've shown $x_0 + x_1$ & $y_0 + y_1$ are both sol's.

Note: We're shown that sol's of this type of system are "closed under scaling (scalar multp.) & addition