

The acceleration of the piston

$$a(\theta) = 2\omega^2(\cos\theta + .5\cos(2\theta))$$



If $\omega = \text{constant}$

Find which angle (what position) gives
max/min acceleration.

$$\begin{aligned} a'(\theta) &= -2\omega^2(\sin\theta + \frac{1}{2}(-\sin(2\theta) \cdot 2)) \\ &= -2\omega^2(\sin\theta - \sin 2\theta) \end{aligned}$$

$\sin 2\theta = 2\cos\theta\sin\theta$

Why?

Basic Fact:

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$A = \theta$$

$$B = \theta$$

$$\sin(2\theta) = \sin\theta \cos\theta + \cos\theta \sin\theta$$

$$a'(\theta) = -2\omega^2(\sin\theta - 2\sin\theta\cos\theta) = 0$$

$$\underbrace{-2\omega^2\sin\theta}_{=0} \underbrace{(1 - 2\cos\theta)}_{=0} = 0$$

$$\frac{-2\omega^2\sin\theta}{-2\omega^2} = 0$$

$$\sin\theta = 0$$

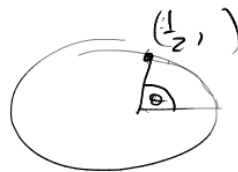
$\theta = 0, \pi$



$$1 - 2\cos\theta = 0$$

$$1 = 2\cos\theta$$

$$\frac{1}{2} = \cos\theta$$



$\theta = \frac{\pi}{3}$

$$y = 4x^2$$

$$x = 0$$

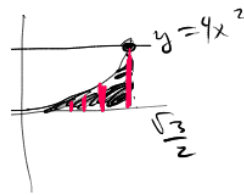
$$x = 3$$

$$y = 3$$

x-axis

$$4x^2 = 3$$

$$x = \pm \frac{\sqrt{3}}{2}$$



1. washer or disc



$$r = 4x^2$$

$$A = \pi r^2$$

$$= \pi 16x^4$$

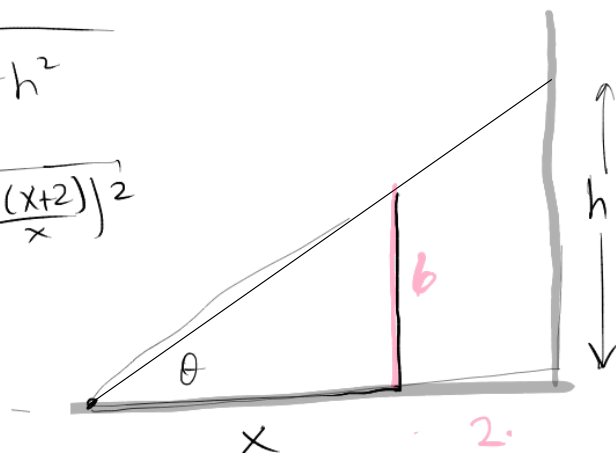
$$\int_0^3 \pi 16x^4 dx = \pi \cdot \frac{16x^5}{5} \Big|_0^3 = \boxed{\pi \frac{16(3)^5}{5}} - \frac{16(0)^5}{5}$$

$$= 0$$

$$l = \sqrt{(x+2)^2 + h^2}$$

$$= \sqrt{(x+2)^2 \left(\frac{6(x+2)}{x} \right)^2}$$

As
 $l(4.1)$
 $= 1$



$$\frac{b}{x} = \frac{h}{x+2}$$

$$\frac{6(x+2)}{x} = h$$

Minimize l .

B/c l is strictly increasing, we can minimize l^2 instead, getting same x .

minimize l^2 instead. : $\left((x+2)^2 + \left(\frac{6(x+2)}{x} \right)^2 \right)' = 0$

$$2(x+2) + 2 \left(\frac{6(x+2)}{x} \right)' \left(\frac{-12}{x^2} \right)$$

$$2(x+2) \left(1 + \frac{6}{x} \left(\frac{-12}{x^2} \right) \right) = 0$$

$\Rightarrow 0$
 $x = -2$

$$1 - \frac{72}{x^3} = 0$$

dist. = 4.
 $x = \sqrt[3]{72} \approx 4.1$