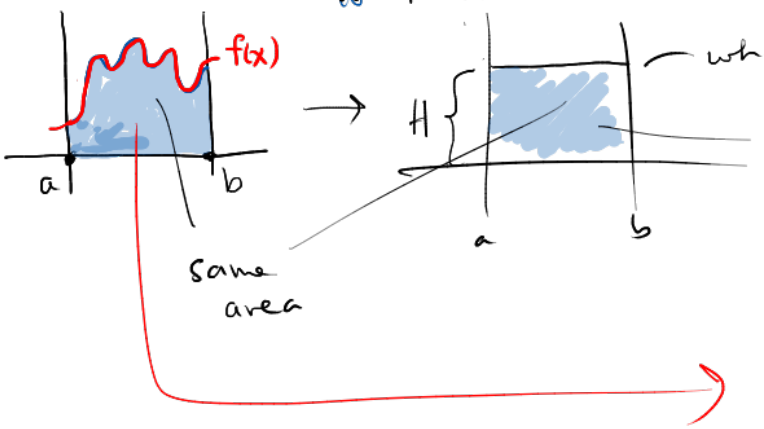
Applications of Calculus

1. Average Value



what is the average depth of water?

what height is this?

Area = H(b−a)

height, width

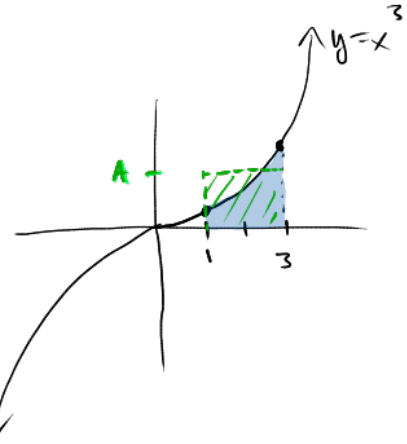
SAME

$$\text{Area} = \int_a^b f(x)\,dx = H(b-a)$$

So the average height is this →

$$\boxed{\left(\frac{1}{b-a}\right)\int_a^b f(x)\,dx = H}$$

$$\int_1^3 x^3\,dx = \frac{x^4}{4}\Big|_1^3 = \frac{81}{4} - \frac{1}{4} = \frac{80}{4} = 20$$

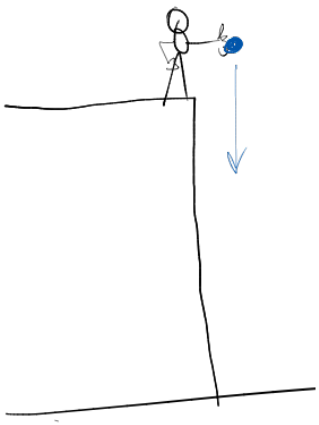


what must A be? = 10.

$$\text{Avg. Value} = \frac{1}{(3-1)}\int_1^3 x^3\,dx = \frac{1}{2}\cdot 20 = 10$$

(We're on earth) gravity = -32 ft/sec^2 (acceleration)

Find the speed of a rock when it hits the ground if it is dropped from 256 feet high.



start: acceleration of the rock at time t.

$t=0 \leftrightarrow$ instant rock is dropped

$a(t) = -32$

velocity is related to acceleration:

$$\frac{d}{dt}(v(t)) = a(t).$$

we need a formula for velocity.

we know the velocity, but we don't know when the rock hits the ground.

$$\int \frac{d}{dt}(v(t)) = \int a(t).$$

$$v(t) = \int a(t)\,dt = \int -32\,dt$$

$$v(t) = -32t + C$$

$v(0) = -32(0) + C$

$0 \Rightarrow C = 0$

$$v(t) = -32t$$

The rock hits the ground when it travels 256 feet.

We need a formula for position

$$s(t) = \int v(t)\,dt$$

position $= \int -32t\,dt = -\frac{32t^2}{2} + C$

$= -16t^2 + C$

position @ time $= 0$

$s(0) = -16(0)^2 + C$

$256 \Rightarrow C = 256$

$$s(t) = -16t^2 + 256$$

ground level $= s(t) = 0$

set $-16t^2 + 256 = 0$ & solve for t

$-16t^2 = -256$

$t^2 = 16 \Rightarrow t = \pm 4$, $t = 4$

$t=4$ when rock hits

$v(4) = -32(4)$

$= -128 \frac{\text{ft}}{\text{sec}}$

Find the velocity of an object whose position is above ground is at $t=5$

$$s(t) = t(t^3+1)^2$$

$$= t(t^6 + 2t^3 + 1)$$

$$s(t) = t^7 + 2t^4 + t$$

$$s'(t) = 7t^6 + 8t^3 + 1$$

$$s'(5) = 7(5)^6 + 8(5)^3 + 1$$

$$= 110,376$$

$$s(t) = t(t^3+1)^2$$

$$s'(t) = 1\cdot(t^3+1)^2 + t\left(2(t^3+1)^1 \cdot 3t^2\right)$$

$$= (t^3+1)^2 + 6t^3(t^3+1)$$