

Good luck on your FINALS!

# Polynomials and Radicals

(a) Derivatives: Compute  $f'(x)$  of the following

i.  $f(x) = 6x^2 - 12x + 1$

$$f'(x) = 12x - 12 = 12(x-1)$$

ii.  $f(x) = 7x^{-2} + 12x^{-3} + 11$

$$f'(x) = -14x^{-3} - 36x^{-4} = -2 \left( \frac{7}{x^3} + \frac{18}{x^4} \right)$$

iii.  $f(x) = 8x^{1/2} - 12x^{-3/2} + 10$

$$f'(x) = 4x^{-1/2} + 18x^{-5/2} = 2 \left( \frac{2}{x^{1/2}} + \frac{9}{x^{5/2}} \right)$$

iv.  $f(x) = 6\sqrt{x} + 12x^{-3/2} - 12$

$$f'(x) = 3x^{-1/2} - 18x^{-5/2} = 3 \left( \frac{1}{x^{1/2}} - \frac{6}{x^{5/2}} \right)$$

v.  $f(x) = (x^3 - 12x)(1 + 2x + 3x^2)$

$$f'(x) = (3x^2 - 12)(1 + 2x + 3x^2) + (x^3 - 12x)(2 + 6x)$$

vi.

$$f(x) = \frac{(2x^2 - 12x)}{(1 + 2x)}$$

$$f'(x) = \frac{(1+2x)(4x-12) - (2x^2-12x)(2)}{(1+2x)^2} = \frac{4x^2 + 4x - 12}{(1+2x)^2}$$

i.  $f(x) = (3x + 1)^4$

$$f'(x) = 4(3x+1)^3 \cdot 3 = 12(3x+1)^3$$

ii.  $f(x) = (1 - 2x)^{-4}$

$$\begin{aligned} f'(x) &= -4(1-2x)^{-5} \cdot (-2) \\ &= 8(1-2x)^{-5} \end{aligned}$$

(a) Integrals: Compute  $\int f(x)dx$

i.  $f(x) = 6x^2 + 12x + 1$  Compute  $\int_1^3 f(x)dx$

$$\begin{aligned}\int f(x) &= \left. \frac{6x^3}{3} + \frac{12x^2}{2} + x \right|_1^3 \\ &= \left. 2x^3 + 6x^2 + x \right|_1^3 \\ &= \underbrace{2 \cdot 27}_{54} + 54 + 3 - (2 + 6 + 1) \\ &= 102\end{aligned}$$

ii.  $f(x) = 7x^{-2} + 12x^{-3} + 11$

$$\begin{aligned}\int f(x) &= \frac{7x^{-1}}{-1} + \frac{12x^{-2}}{-2} + 11x + C \\ &= -\frac{7}{x} - \frac{6}{x^2} + 11x + C\end{aligned}$$

iii.  $f(x) = 8x^{1/2} - 12x^{-3/2} + 10$

$$\begin{aligned}\int f(x) dx &= \frac{2}{3} \cdot 8x^{3/2} + \frac{2}{1} \cdot 12x^{-1/2} + 10x + C \\ &= \frac{16}{3}x^{3/2} + 24x^{-1/2} + 10x + C\end{aligned}$$

i.  $f(x) = (2x+1)^4$

$$u = 2x+1$$

$$\frac{du}{dx} = 2 \Rightarrow du = 2dx$$

$$\Rightarrow \frac{du}{2} = dx$$

$$\int (2x+1)^4 dx = \int u^4 \cdot \frac{du}{2} = \frac{1}{2} \int u^4 du = \frac{1}{2} \frac{u^5}{5} + C$$

$$= \frac{1}{10} (2x+1)^5 + C$$

ii.  $f(x) = \sqrt{3x+1}$

$$\int_1^3 (3x+1)^{1/2} dx = \int u^{1/2} \cdot \frac{1}{3} du = \frac{1}{3} \int u^{1/2} du = \frac{1}{3} \frac{u^{3/2}}{3/2}$$

$$u = 3x+1$$

$$\frac{du}{dx} = 3 \Rightarrow du = 3dx$$

$$\frac{1}{3} du = dx$$

$$= \frac{2}{9} (3x+1)^{3/2} \Big|_1^3$$

$$= \frac{2}{9} \left[ (10)^{3/2} - (4)^{3/2} \right]$$

iii.  $f(x) = x(3x^2+1)$

$$u = 3x^2+1$$

$$\frac{du}{dx} = 6x \Rightarrow du = 6x dx \Rightarrow \frac{du}{6} = x dx$$

$$\int x(3x^2+1) dx = \int (3x^2+1)x dx = \int u \cdot \frac{du}{6} = \frac{1}{6} \int u du$$

$$= \frac{1}{6} \frac{u^2}{2} + C$$

or distribute 1st

$$\int 3x^3 + x dx = \frac{3x^4}{4} + \frac{x^2}{2} + C = \frac{3x^4 + 2x^2}{4} + C$$

$$= \frac{(3x^2+1)^2}{12} + C$$

Page 6

How can both of these be right?

$$\frac{(3x^2+1)^2}{12} + C = \frac{9x^4 + 6x^2 + 1}{12} + C = \frac{3x^4 + 2x^2}{4} + \underbrace{\frac{1}{12}}_{\text{constant}} + C$$

Two anti-derivatives are equal if they differ only by a constant.

(a) Applications

- i. Find the speed of a pebble the instant it hits the ground if it is dropped from 256 feet high.

$$v(t) = \int -32 dt = -32t + C, \quad \frac{1}{2} C = 0 \quad (\text{since dropped})$$

$$s(t) = \int -32t dt = -16t^2 + C$$

$$\frac{1}{2} s(0) = 256 \Rightarrow C = 0 \quad \text{so} \quad s(t) = -16t^2 + 256$$

$$\text{Pebble hits ground: } s(t) = 0 = -16t^2 + 256$$

$$\Rightarrow t^2 = \frac{256}{16} = \frac{2^8}{2^4} = 2^4 = 16$$

$$\text{so } t = 4 \text{ seconds}$$

Velocity upon impact:

$$v(4) = -32(4) = -128 \text{ ft/sec}$$

- ii. Find the velocity at  $t = 2$  of an object whose position is  $s(t) = x(3x+1)^2$  feet high at time  $t$  (in seconds).

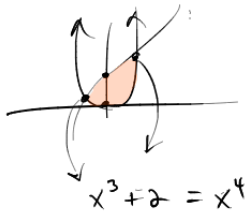
$$v(t) = s'(t) = 1 \cdot (3x+1)^2 + x \cdot (2(3x+1) \cdot 3)$$

$$= (3x+1)^2 + 6x(3x+1) = (3x+1)((3x+1) + 6x)$$

$$= (3x+1)(9x+1)$$

$$v(2) = (3 \cdot 2 + 1)(9 \cdot 2 + 1) = (7)(19) = \boxed{133 \frac{\text{ft}}{\text{sec}}}$$

Find the area bound by  $y = x^3 + 2$  &  $y = x^4$ .

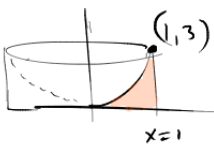


$x^4 - x^3 + 2 = 0$  tough to solve by hand look at graph instead

$\Rightarrow x = -1 \quad \frac{1}{4} x = 1.54$

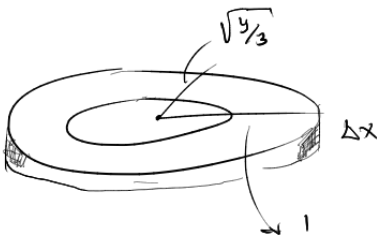
$$A = \int_{-1}^{1.54} (x^3 + 2 - x^4) dx = \left[ \frac{x^4}{4} + 2x - \frac{x^5}{5} \right]_{-1}^{1.54} = \frac{(1.54)^4}{4} + 2(1.54) - \frac{(1.54)^5}{5} - \left( \frac{1}{4} - 2 + \frac{1}{5} \right) \approx 4.03$$

(iv) Find the volume:  $y = 3x^2$ ,  $y = 0$ ,  $x = 0$ ,  $x = 1$ , about  $y$ -axis



$y$ -axis  $\Rightarrow \frac{y}{3} = x^2$  bounds  $\downarrow$   $y = 3 \cdot 0^2 = 0$   $y = 3 \cdot 1^2 = 3$

$\Rightarrow \sqrt{\frac{y}{3}} = x$  will be inside (small) radius

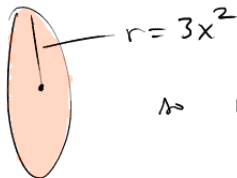


$$A = \pi(1)^2 - \pi(\sqrt{y/3})^2$$

$$V = \int_0^3 \pi(1 - y/3) dy = \pi \left( y - \frac{y^2}{6} \right) \Big|_0^3 = \pi \left( 3 - \frac{9}{6} \right)$$

$$= \boxed{\frac{9\pi}{2}}$$

(v) Now the  $x$ -axis



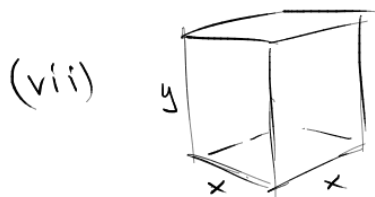
$\Rightarrow A = \pi(3x^2)^2 = 9\pi x^4$

$$\int_0^1 9\pi x^4 dx = \left. \frac{9\pi}{5} x^5 \right|_0^1 = \boxed{\frac{9\pi}{5}}$$

(vi) Find the average velocity,  $v(t) = \cos(2t)$ ,  $t \in [0, 2\pi]$

$$= \frac{1}{2\pi - 0} \int_0^{2\pi} \cos(2t) = \frac{1}{2} \frac{1}{2\pi} (-\sin(2t)) \Big|_0^{2\pi} = 0.$$

The projectile is oscillating forward (+ velocity) and back (- velocity) equally, giving an average velocity of zero.



1000 cm<sup>2</sup> of material

constraint:

$$\left. \begin{array}{l} 1 \text{ Bottom: } x^2 \\ 4 \text{ sides: } xy \end{array} \right\} \begin{array}{l} x^2 + 4xy = 1000 \\ y = \frac{1000 - x^2}{4x} \end{array}$$

$$A = x^2 y = x^2 \left( \frac{1000 - x^2}{4x} \right)$$

$$= \frac{1000x - x^3}{4} = 250x - .25x^3$$

$$A' = 250 - .75x^2 = 0$$

$$250 = .75x^2$$

$$\frac{250}{.75} = x^2$$

$$\frac{4 \cdot 250}{3} = x^2$$

$$\frac{2 \cdot 5 \sqrt{10}}{\sqrt{3}} = x$$

$$10 \cdot \sqrt{\frac{10}{3}} = x$$

$$\frac{1000 - (18.25)^2}{4(18.25)}$$

$$18.25 \approx x$$

$$9.13 \approx y$$

Note!  $A'' = -1.5x$

$$\frac{1}{3} A''(18.25) < 0$$

$\Rightarrow$   $\cap$  concave down

$\Rightarrow$  volume is a max.

25.13

### 3. Exponentials

(a) Derivatives: Compute  $f'(x)$  of the following

i.  $f(x) = e^{2x+1}$  Compute  $\int_1^3 f(x)dx$

$$f'(x) = 2e^{2x+1} \Big|_1^3 = 2[e^7 - e^3] \approx 2153.1$$

ii.  $f(x) = e^{-3x^2+x}$

$$f'(x) = e^{-3x^2+x} (-6x+1)$$

iii.  $f(x) = 2e^{\cos(x)}$

$$f'(x) = 2e^{\cos(x)} \cdot (-\sin(x)) = -2\sin(x)e^{\cos(x)}$$

iv.  $f(x) = xe^x$

$$f'(x) = e^x + xe^x = e^x(1+x)$$

v.  $f(x) = 2xe^{3x}$

$$f'(x) = 2e^{3x} + \underline{3 \cdot 2x e^{3x}} = 2e^{3x}(1 + 3x)$$

+ C

vi.

$$f(x) = \frac{2x+1}{e^{3x}}$$

$$\begin{aligned} f'(x) &= \frac{2e^{3x} - (2x+1)e^{3x} \cdot \frac{1}{3}}{e^{6x}} = \frac{e^{3x} \left( 2 - \frac{(2x+1)}{3} \right)}{e^{6x}} \\ &= \frac{6 - (2x+1)}{e^{3x}} = \frac{5-2x}{e^{3x}} \end{aligned}$$



(a) Integrals

$$u = 2x \\ \frac{du}{dx} = 2 \quad du = 2dx \quad \frac{du}{2} = dx$$

i.  $f(x) = e^{2x}$

$$\int e^{2x} dx = \int e^u \frac{du}{2} = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C \\ = \frac{1}{2} e^{2x} + C$$

ii.  $f(x) = e^{-3x}$

$$u = -3x \\ \frac{du}{dx} = -3 \\ \Rightarrow du = -3dx \\ -\frac{1}{3} du = dx$$
$$\int e^{-3x} dx = \int e^u \left(-\frac{1}{3} du\right) = -\frac{1}{3} \int e^u du = -\frac{1}{3} e^u + C \\ = -\frac{1}{3} e^{-3x} + C$$

i.  $f(x) = e^{2x+1}$

$$u = 2x + 1 \\ \frac{du}{dx} = 2 \\ \Rightarrow du = 2dx \\ \frac{1}{2} du = dx$$
$$\int e^{2x+1} dx = \int e^u \left(\frac{1}{2} du\right) = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C \\ = \frac{1}{2} e^{(2x+1)} + C$$

ii.  $f(x) = x e^{2x^2+1}$

$$u = 2x^2 + 1 \\ \frac{du}{dx} = 4x \\ \Rightarrow du = 4x dx \Rightarrow \frac{du}{4} = x dx$$
$$\int x e^{2x^2+1} dx = \int e^u \cdot \frac{du}{4} = \frac{1}{4} \int e^u du = \frac{1}{4} e^u + C \\ = \frac{1}{4} e^{(2x^2+1)} + C$$

iii.  $f(x) = \cos(x) e^{\sin(x)}$

$$u = \sin x \\ \frac{du}{dx} = \cos x \\ \Rightarrow du = \cos x dx$$
$$\int \cos x \cdot e^{\sin x} dx = \int e^u \underbrace{\cos x dx}_{du} = \int e^u du \\ = e^u + C \\ = e^{\sin x} + C$$

iv.  $f(x) = \sin(x) e^{2\cos(x)+1}$

$$u = 2\cos x + 1$$

$$\frac{du}{dx} = -2\sin x$$

$$du = -2\sin x dx$$

$$\Rightarrow -\frac{1}{2} du = \sin x dx$$

$$\int e^{2\cos x + 1} \cdot \underbrace{\sin x dx}_{-\frac{1}{2} du} = \int e^u \cdot -\frac{1}{2} du \\ = -\frac{1}{2} \int e^u du = -\frac{1}{2} e^u + C \\ = -\frac{1}{2} e^{2\cos(x)+1} + C$$

$$u = x \quad dv = e^x$$

$$\frac{du}{dx} = 1 \quad v = e^x$$

i.  $f(x) = xe^x$

$$\int xe^x = xe^x - \int e^x dx = xe^x - e^x + C$$

$$= e^x(x-1) + C$$

ii.  $f(x) = -xe^{2x}$

$$u = -x \quad dv = e^{2x}$$

$$\frac{du}{dx} = -1 \quad v = \frac{1}{2}e^{2x}$$

$$\left. \begin{array}{l} u = -x \\ dv = e^{2x} \end{array} \right\} \begin{array}{l} -\frac{x}{2}e^{2x} + \frac{1}{2}\int e^{2x} dx \\ -\frac{x}{2}e^{2x} + \frac{1}{4}e^{2x} + C \end{array}$$

$$-\frac{e^{2x}}{2} \left( x - \frac{1}{2} \right) + C$$

iii.  $f(x) = 5xe^{-2x}$

$$u = 5x \quad dv = e^{-2x}$$

$$\frac{du}{dx} = 5 \quad v = -\frac{1}{2}e^{-2x}$$

$$du = 5 dx$$

$$\left. \begin{array}{l} u = 5x \\ dv = e^{-2x} \end{array} \right\} \begin{array}{l} \Rightarrow -\frac{5x}{2}e^{-2x} + \frac{5}{2}\int e^{-2x} dx \\ -\frac{5x}{2}e^{-2x} + \frac{5}{2} \cdot \frac{1}{2}e^{-2x} + C \end{array}$$

$$-\frac{5}{2}e^{-2x} \left( x + \frac{1}{2} \right) + C$$

(a) Applications

- i. Find the velocity at  $t = 5$  of an object moving according to  $s(t) = x(e^{-x})$

$$s'(t) = v(t) = 1 \cdot e^{-x} - x e^{-x} = e^{-x}(1-x)$$

$$v(5) = e^{-5}(1-5) = \frac{-4}{e^5} \approx -0.03$$

Ignore Gravity!

- ii. Find the height at  $t = 10$  of a bottle rocket whose velocity is  $v(t) = x(e^{-x})$ , where  $t = 0$  corresponds to the moment of liftoff.

$$s(t) = \int x e^{-x} dx = -x e^{-x} + \int e^{-x} dx = -x e^{-x} - e^{-x} + C$$
$$\left. \begin{array}{l} u = x \quad dv = e^{-x} \\ \frac{du}{dx} = 1 \quad v = -e^{-x} \end{array} \right\} \quad \frac{1}{2} s(0) = 0 \Rightarrow \underbrace{-0e^{-0}}_0 - \underbrace{e^{-0}}_{-1} + C = 0 \Rightarrow C = 1$$

$$s(t) = 1 - e^{-x}(x+1)$$

$$s(10) = 1 - e^{-10}(10+1) = 1 - \frac{11}{e^{10}} \approx .999$$

- iii. Kyle, who likes guns, fires a bullet straight up into the air. How high does the bullet go? Ignore air resistance, assume the initial velocity of the bullet is ~~400~~ 400 feet per second, and assume gravity is  $-32 \frac{\text{feet}}{\text{sec}^2}$ .

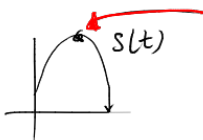
$$a(t) = -32$$

$$v(t) = -32t + C \quad \frac{1}{2} v(0) = 400 \Rightarrow C = 400$$

$$v(t) = -32t + 400$$

$$s(t) = -16t^2 + 400t + C, \text{ assume } s(0) = 6 \text{ (Kyle's 6 feet tall)}$$

$$s(t) = -16t^2 + 400t + 6$$



$$s'(t) = -32t + 400 = 0$$

$$t = \frac{400}{32} = \frac{100}{8} = \frac{25}{2} = 12.25 \text{ s.} \Rightarrow s(12.25) = 2505 \text{ feet.}$$

#### 4. Logarithmic Functions

(a) Derivatives: Compute  $f'(x)$  of the following

i.  $f(x) = \ln(\cos(x))$

$$f'(x) = \frac{-\sin x}{\cos x} = -\tan x$$

ii.  $f(x) = \ln(2x)$

$$f'(x) = \frac{2}{2x} = \frac{1}{x}$$

iii.  $f(x) = x \ln(x)$

$$f'(x) = \ln x + \frac{x}{x} = 1 + \ln x$$

iv.  $f(x) = 2x \ln(3x)$

$$f'(x) = 2 \ln(3x) + 2x \cdot \frac{3}{3x} = 2 \ln 3x + 2$$

v.

$$f(x) = \frac{\ln x}{3x}$$

$$f'(x) = \frac{3x \left(\frac{1}{x}\right) - 3 \ln x}{9x^2} = \frac{3(1 - \ln x)}{9x^2} = \frac{1 - \ln x}{3x^2}$$

vi.

$$f(x) = \frac{2x}{\ln x + 1}$$

$$f'(x) = \frac{2(\ln x + 1) - 2x \left(\frac{1}{x}\right)}{(\ln x + 1)^2} = \frac{2 \ln x}{(\ln x + 1)^2}$$

(a) Integrals

i.

$$f(x) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \ln(x) + c$$

ii.

$$f(x) = \frac{\sin(x)}{\cos(x)}$$

$$u = \cos x$$

$$\frac{du}{dx} = -\sin x$$

$$-du = \sin x dx$$

$$\int \frac{\sin x}{\cos x} dx = \int -\frac{du}{u}$$

$$= -\ln u + c$$

$$= -\ln(\cos x) + c$$

iii.

$$f(x) = \frac{x^2}{x^3 + 1}$$

$$u = x^3 + 1$$

$$\frac{du}{dx} = 3x^2$$

$$du = 3x^2 dx$$

$$\frac{1}{3} du = x^2 dx$$

$$\int \frac{x^2}{x^3 + 1} dx = \int \frac{\frac{1}{3} du}{u} = \frac{1}{3} \int \frac{du}{u} = \frac{1}{3} \ln u + c$$

$$= \frac{1}{3} \ln(x^3 + 1) + c$$

(a) Applications

- i. Suppose the height of an object is given by  $f(x) = \ln(x)$ . Which is greater, the speed of the object at  $t = 10$  or  $t = 100$ ? Is the object going up or down? Is the object accelerating or decelerating at these times?

$$v(x) = \frac{1}{x} \quad \text{so} \quad v(10) = \frac{1}{10} > v(100) = \frac{1}{100}$$

$t=10 \Rightarrow$  greater speed, it's positive, so going up

$$a(x) = -\frac{1}{x^2}, \text{ negative, decelerating always.}$$

## 5. Trigonometric Functions

(a) Derivatives: Compute  $f'(x)$  of the following

i.  $f(x) = \cos 2x$

$$f'(x) = -2\sin 2x$$

ii.  $f(x) = \tan x = \frac{\sin x}{\cos x}$

$$f'(x) = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

iii.  $f(x) = \sin(3x)e^{2x}$

$$f'(x) = 3\cos(3x)e^{2x} + 2\sin(3x)e^{2x}$$

iv.

$$f(x) = \frac{1}{\sin x}$$

$$f'(x) = \frac{-\cos x}{\sin^2 x}$$

v.  $f(x) = \cos 2x^3 + x$

$$f'(x) = -\sin(2x^3) \cdot (6x^2) + 1$$

vi.  $f(x) = \sin e^{2x}$

$$f'(x) = \cos(e^{2x}) \cdot e^{2x} \cdot 2$$

vii.  $f(x) = (1 + \sin 2x)^3$

$$f'(x) = 6(1 + \sin 2x)^2 (\cos 2x)$$

viii.  $f(x) = x \cos x$

$$f'(x) = \cos x - x \sin x$$

(a) Integrals

i.  $f(x) = \sin 2x$

$$\int \sin 2x \, dx = \int \sin u \cdot \frac{1}{2} du = \frac{1}{2} \int \sin u \, du$$

$$u = 2x$$

$$\frac{du}{dx} = 2$$

$$du = 2 dx$$

$$\frac{1}{2} du = dx$$

$$= -\frac{1}{2} \cos u + C$$

$$= -\frac{1}{2} \cos(2x) + C$$

ii.  $f(x) = \cos(-2x)$

$$u = -2x$$

$$\frac{du}{dx} = -2$$

$$du = -2 dx$$

$$-\frac{1}{2} du = dx$$

$$\int \cos(-2x) \, dx = \int \cos(u) \left(-\frac{1}{2} du\right)$$

$$= -\frac{1}{2} \int \cos u \, du$$

$$= -\frac{1}{2} \sin u + C$$

$$= -\frac{1}{2} \sin(-2x) + C$$



i.  $f(x) = x \cos(-2x^2)$

$$u = -2x^2$$

$$\frac{du}{dx} = -4x$$

$$du = -4x dx$$

$$-\frac{1}{4} du = x dx$$

$$\int \cos(-2x^2) x dx$$

$$= \int \cos(u) \left(-\frac{1}{4} du\right)$$

$$= -\frac{1}{4} \int \cos u du = -\frac{1}{4} \sin u + C$$

$$= -\frac{1}{4} \sin(-2x^2) + C$$

ii.  $f(x) = x^2 \sin(-2x^3)$

$$u = -2x^3$$

$$\frac{du}{dx} = -6x^2$$

$$du = -6x^2 dx$$

$$-\frac{1}{6} du = x^2 dx$$

$$\int \underbrace{\sin(-2x^3)}_u \underbrace{x^2 dx}_{-\frac{1}{6} du} = \int \sin(u) \left(-\frac{1}{6} du\right)$$

$$= -\frac{1}{6} \int \sin u du$$

$$= +\frac{1}{6} \cos u + C$$

$$\frac{1}{6} \cos(-2x^3) + C$$

iii.  $f(x) = \sin(x)(1 + \cos x)^3$

$$u = 1 + \cos x$$

$$\frac{du}{dx} = -\sin x$$

$$du = -\sin x dx$$

$$-du = \sin x dx$$

$$\int (1 + \cos x)^3 \sin x dx$$

$$\int u^3 (-du) = -\int u^3 du$$

$$= -\frac{u^4}{4} + C$$

$$= -\frac{(1 + \cos x)^4}{4} + C$$

i.  $f(x) = x \sin(x)$

$$\int x \sin(x) dx = -x \cos(x) + \int \cos x dx$$

$$u = x \quad dv = \sin x dx \quad = -x \cos x + \sin x + C$$

$$\frac{du}{dx} = 1 \quad v = -\cos x$$

$$du = dx$$

ii.  $f(x) = 2x^2 \cos(3x)$

$$\left. \begin{array}{l} u = 2x^2 \quad dv = \cos(3x) \\ du = 4x \quad v = \frac{1}{3} \sin 3x \end{array} \right\}$$

$$\left. \begin{array}{l} u = x \quad dv = \sin 3x \\ du = 1 \quad v = -\frac{1}{3} \cos 3x \end{array} \right\}$$

$$\frac{2x^2}{3} \sin 3x + \frac{4}{3} \int x \cdot \sin 3x dx$$

$$+ \frac{4}{3} \left( -\frac{x}{3} \cos 3x + \frac{1}{3} \int \cos 3x dx \right)$$

$$+ \frac{4}{3} \left( -\frac{x}{3} \cos 3x + \frac{1}{3} \cdot \frac{1}{3} \sin 3x \right)$$

$$\frac{2x^2}{3} \sin 3x - \frac{4x}{9} \cos 3x + \frac{4}{27} \sin 3x$$

$$\frac{2}{3} \left[ \sin 3x \left( x^2 + \frac{2}{9} \right) - \frac{4}{3} x \cos 3x \right] + C$$

(a) Applications

- i. Find the velocity at time  $t = 3$  of an object whose position is

$$s(t) = 2\cos(1.3t - 2)$$

---

$$s'(t) = -2\sin(1.3t - 2)(1.3)$$

$$s'(3) = -2\sin(1.3(3) - 2)(1.3) = -2.46$$

- ii. Find the acceleration at time  $t = 10$  of an object whose position is given by

$$s(t) = 2\cos(1.3t - 2)$$

$$v(t) = -2\sin(1.3t - 2) \cdot (1.3) = -2.6\sin(1.3t - 2)$$

$$a(t) = -2.6\cos(1.3t - 2)(1.3)$$

$$= -3.38\cos(1.3t - 2)$$

$$a(10) = -3.38\cos(13 - 2) = -3.38\cos(11)$$

$$= -0.015$$