Exam 4 Guide :: Math 271 :: April 23, 2017

GOOD LUCK ON YOUR EXAMS!
- DR. THOMPSON

1. Jared throws a rock down from the roof of a building which is 100 feet tall. The initial velocity of the ball is 20 $\frac{ft}{sec}$, the effect of air resistance is negligible and the effect of gravity is $-32\frac{ft}{sec}$.

$V(0) = -20$ ← initial velocity

$a(t) = -32$

$V(t) = \int a(t)\,dt = -32t + C$

$V(0) = -32(0) + C$ so $C = -20$

$V(t) = -32t - 20$

When does the ball strike the ground?

When is the position fcn $= 0$?

posit $= s(t) = \int v(t)\,dt = \int -32t - 20\,dt = -\frac{32t^2}{2} - 20t + C$

$s(0) = -\frac{32(0)^2}{2} - 20(0) + C \Rightarrow C = 100$

$s(t) = -16t^2 - 20t + 100$

height of ball above ground

solve $s(t) = 0$

$-16t^2 - 20t + 100 = 0$

$4t^2 + 5t - 25 = 0$

$t = \frac{-5 \pm \sqrt{25 - 4(4)(-25)}}{2(4)}$

$= \frac{-5 \pm \sqrt{425}}{8}$

$= \frac{-5 \pm 20.6}{8}$

$= \frac{15.6}{8}$ or $\frac{-25.6}{8}$

1.95 s

With what speed does the ball strike the ground?

$V(1.95) = -32(1.95) - 20 = -82.4 \frac{ft}{sec}$

2. Sierra is playing with a yo-yo. The yo-yo is moving at a velocity of

$$v(t) = -.2\sin(4t)$$

feet per second where velocity is positive if the yo-yo is above her knee and $v(t)$ is negative if the yo-yo is below her knee.

If the yo-yo is 1 foot below her knee at time $t = \pi/4$, determine if her the yo-yo is above or below her knee at time $t = 1.4$ and $t = 2.1$.

Need $s(t) = \int v(t)\,dt = \int -.2\sin(4t)\,dt = -.2\int \sin(4t)\,dt = -.2\int \sin(u)\frac{du}{4}$

$u = 4t$, $du = 4dt$, $\frac{du}{4} = dt$

$= \frac{-.2}{4}\int \sin u\,du$

$= \frac{-.2}{4}(-\cos(u)) + C$

$s(t) = \frac{1}{20}\cos(4t) + C$

$s(1.4) > 0$ or $s(1.4) < 0$

$\Rightarrow$ positive $\Rightarrow$ above knee

Is her yo-yo going up or down at $t = 1.4$ and $t = 2.1$?

$v(1.4) > 0 \Rightarrow$ up

$v(2.1) < 0 \Rightarrow$ down

$s(\frac{\pi}{4}) = \frac{1}{20}\cos(\frac{4\pi}{4}) + C$

$-1 = \frac{1}{20}\cos(\pi) + C$

$-1 = \frac{1}{20}(-1) + C \Rightarrow C = 1\frac{1}{20} = \frac{21}{20}$ ft

3. Suppose the number of errors e made by a manufactoring system is proportional to the number of hours t the system has been running since reset and the data suggests that the number of errors made <u>at</u> time t is

$e = 3t$.

$$\int_0^{10} 3t\,dt = \frac{3t^2}{2}\Big|_0^{10} = \frac{3(100)}{2} = 3 \cdot 50 = 150$$

Find the <u>total</u> number of errors made by the system if it has been running for 10 hours.

$\int dt = \infty$ additions

$= 150$

Find the average number of errors made per hour.

$$\text{Average Value} = \frac{1}{b-a}\int_a^b f(t)\,dt$$

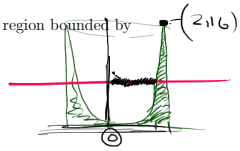
$$\frac{1}{10-0}\int_0^{10} 3t\,dt$$

$$= \frac{1}{10-0} \cdot 150 = 15 \text{ errors per hour}$$

4. Find the volume of the solid obtained by rotating the region bounded by

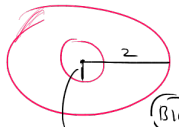
$y = x^4, y = 0, x = 0, x = 2$

about the y-axis.



revolving about x-axis $\Rightarrow$ Integrate w.r.t. x
or something parallel to the axis $(y = -2)$

revolve about y-axis $\Rightarrow$ Int. w.r.t y
$\Rightarrow$ get func in terms of y.



BIG − SMALL

$A = \pi(2)^2 - \pi(\sqrt{y})^2$

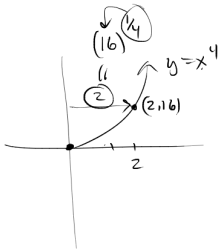
$= 4\pi - y\pi$

$y = x^4 \Rightarrow (y)^{1/4} = x$

$$\Rightarrow V = \int_0^{16} 4\pi - y^{\frac{1}{2}}\pi\,dy$$

$$= \pi\left(4y - \frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right)\Bigg|_0^{16} = \pi\left(64 - 42.\bar{6}\right)$$

$= 21.\bar{3}\pi$ cubic units



$(2^4)^{\frac{1}{2}} =$

$\frac{2}{3}\sqrt{16^3} = \frac{2}{3}\left(\sqrt{16}\cdot\sqrt{16}\cdot\sqrt{16}\right)$
$= \frac{2}{3}(4 \cdot 4 \cdot 4)$
$= \frac{2}{3}64$
$= \frac{128}{3}$
$= 42.\bar{6}$

You try.

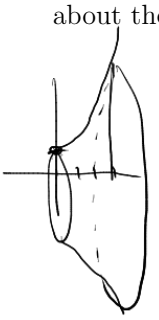
$y = x^2$ instead,

about y-axis And x-axis

5. Find the volume of the solid obtained by rotating the region bounded by

$$y = x^2 + 1, y = 0, x = 0, x = 3$$

about the x-axis.



$$V = \int_0^3 \pi (x^2+1)^2 \, dx$$

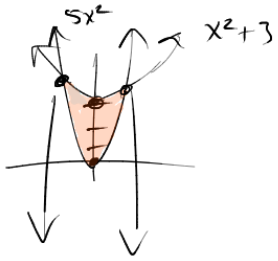
$$= \int_0^3 \pi (x^4 + 2x^2 + 1) \, dx$$

$$= \pi \left(\frac{x^5}{5} + \frac{2x^3}{3} + x \right) \Big|_0^3$$

$$= \pi \left(\frac{3^5}{5} + \frac{2 \cdot 3^3}{3} + 3 \right)$$

$$= \boxed{218.65}$$

6. Find the area bound by $y = 5x^2$ and $y = x^2 + 3$. Sketch the region.



$$A = \int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} x^2 + 3 - 5x^2 \, dx = \int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} -4x^2 + 3 \, dx$$

$5x^2 = x^2 + 3$

$4x^2 = 3$

$x^2 = \pm\frac{\sqrt{3}}{2}$

$$= -\frac{4x^3}{3} + 3x \Big|_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}}$$

$$= 2.89$$

7. Let A be the area of a circle with radius r. If $\frac{dr}{dt} = 2$, find $\frac{dA}{dt}$ when $r = 3$. Explain what this means in terms of rates of change.

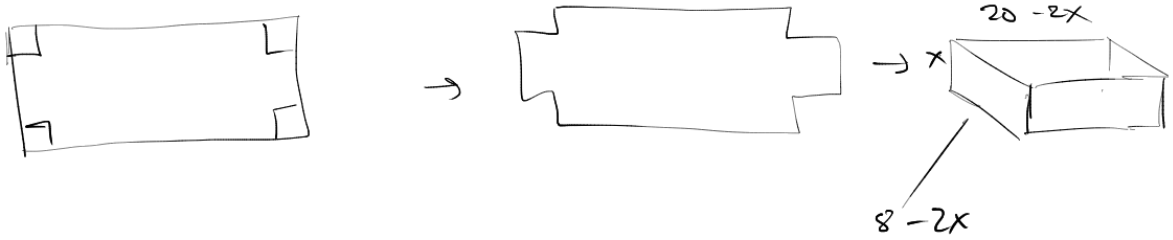
$A = \pi r^2$.

$\frac{dA}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt}$

$= \pi \cdot 2 \cdot 3 \cdot 2 = 12\pi$

the area is increasing at a rate of 12π sq-units per unit of time

8. An open box is to be made out of a 8-inch by 20-inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. Find the dimensions of the resulting box that has the largest volume.



$V = x(8-2x)(20-2x)$

$= (8x - 2x^2)(20 - 2x) = 160x - 16x^2 - 40x^2 + 4x^3$

$V' = 12x^2 - 80x - 32x + 160 = 0$

$= 12x^2 - 112x + 160 = 0 \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = 1.76$ since the other solution forces $8 - 2x < 0$.