Chain Rule: (applies when taking derivative of composition)

M: ~~10~~ ~~11~~ 12

T: ~~8~~ 12

Review: { Tues @ 12 pm watch email for location. }

$$f(x) = \cos\left(e^{\sin(x)}\right)$$

derivative of inside

$$f'(x) = -\sin\left(e^{\sin x}\right) \cdot \left(e^{\sin x} \cdot \cos(x)\right)$$

deriv. of outside

never change inside

$$= -\sin\left(e^{\sin x}\right) \cdot \left(e^{\sin x} \cdot \cos(x)\right)$$

$x^{1/2} \longrightarrow \frac{1}{2}(x)^{-1/2} \cdot 1$

$x \rightarrow 1$

Ex

$$f(x) = \sqrt{3x^2+1} = (3x^2+1)^{\frac{1}{2}}$$

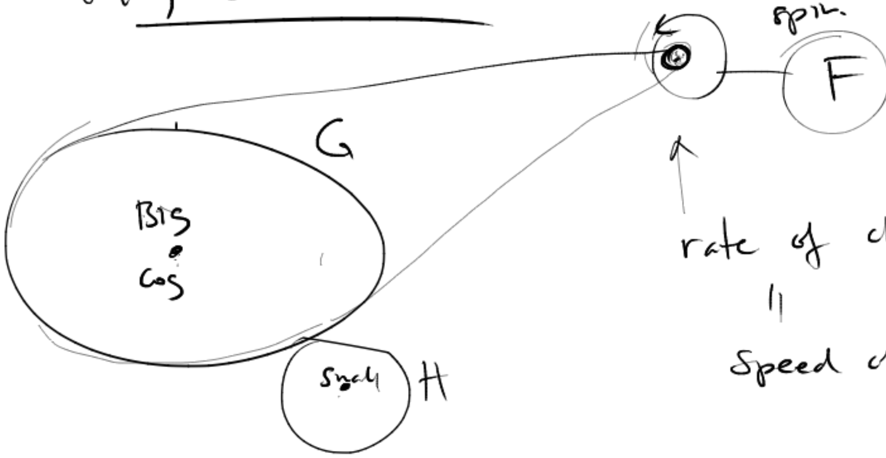
$$f'(x) = \frac{1}{2}(3x^2+1)^{-1/2} \cdot 6x$$

$$f(x) = \frac{1}{\sqrt{5x+x^2}} = \frac{1}{(5x+x^2)^{1/2}} = (5x+x^2)^{-1/2}$$

$f'(x) =$ exponent comes down in front, recopy base, decrease exponent by 1, multiply by derivative of base

$$= -\frac{1}{2}(5x+x^2)^{-3/2} \cdot (5+2x)$$

why chain rule?



rate of change (speed)

=

speed of big cog.

depends on small

$$\frac{dF}{dt} = \frac{dG}{dH} \cdot \frac{dH}{dt}$$

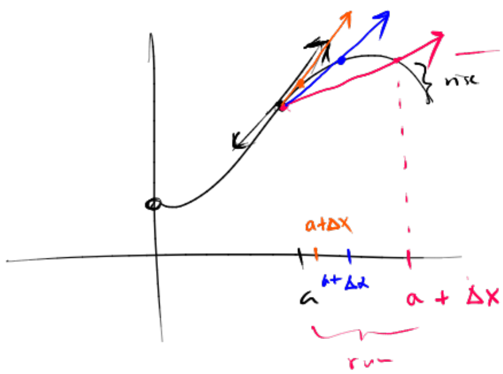
deriv. of out

deriv. of in

What's the def'n of the derivative?

we want to know the slope of the curve at $x=a$

these slopes approach what we want.



$$\boxed{\lim_{\Delta x \to 0} \frac{f(a+\Delta x) - f(a)}{\Delta x}}$$

$h = \Delta x$

Find $f'(x)$ if $f(x) = 3x^2$ using limit def.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{3(x+h)^2 - 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{\boxed{3x^2} + 6xh + 3h^2 \boxed{- 3x^2}}{h}$$

$f(x+h) =$

$$3(x+h)^2 = 3\left(x^2 + 2xh + h^2\right)$$

$$= \lim_{h \to 0} \frac{6xh + 3h^2}{h} = \frac{\cancel{h}(6x + 3h)}{\cancel{h}}$$

$$\lim_{h \to 0} 6x + 3h = 6x$$

derivative using def

$f(x) = \frac{1}{x}$

$$\lim_{h\to 0} \frac{\frac{(x)}{(x)}\frac{1}{x+h} - \frac{1}{x}\frac{(x+h)}{(x+h)}}{h} = \lim_{h\to 0} \frac{\frac{x}{x(x+h)} - \frac{(x+h)}{x(x+h)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{x - (x+h)}{x(x+h)}}{h}$$

$$= \lim_{h\to 0} \frac{\left[\frac{\overbrace{x - x}^{0} - h}{x(x+h)}\right]}{h}$$

$$= \lim_{h\to 0} \frac{-\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}}$$

$$= \lim_{h\to 0} \frac{-1}{x(x+h)} = \frac{-1}{x(x+0)}$$

$$= \frac{-1}{x^2}$$

$$\frac{1}{x} \xrightarrow{d/dx} -\frac{1}{x^2}$$

$$\qquad\qquad\quad \parallel$$

$$x^{-1} = -x^{-2}$$

u-sub:

$$\int e^{2x}\,dx = \int e^{u}\,\frac{du}{2} = \frac{1}{2}\int e^{u}\,du$$

$u = 2x$

$\frac{du}{dx} = 2 \Rightarrow du = 2\,dx$

$\frac{du}{2} = dx$

$= \frac{1}{2}e^{u} + c$

$= \boxed{\frac{1}{2}e^{2x} + c}$

#1: we know $\int e^{u}\,du = e^{u} + c$

we don't know: $\int e^{u}\cdot y \cdot du = ???$

$$\int \frac{x}{\sqrt{x^2+1}}\,dx = \int \frac{x}{(x^2+1)^{1/2}}\,dx$$

$$= \int \underbrace{(x^2+1)}_{u}{}^{-1/2}\cdot \underbrace{x\,dx}_{\frac{du}{2}}$$

$n = -1/2$

$u = x^2+1$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$$\int u^{-1/2}\,\frac{du}{2} = \frac{1}{2}\int u^{-1/2}\,du$$

$= \frac{1}{2}\left(\frac{u^{1/2}}{1/2} + c\right.$

$= u^{1/2} + c$

$= (x^2+1)^{1/2} + c$

$\boxed{\sqrt{x^2+1} + c}$

check:

$$\frac{d}{dx}\left(\sqrt{x^2+1}\right) = \frac{d}{dx}(x^2+1)^{\frac{1}{2}}$$

$$= \frac{1}{2}(x^2+1)^{-\frac{1}{2}}\cdot 2x$$

$$= \frac{x}{\sqrt{(x^2+1)}}$$