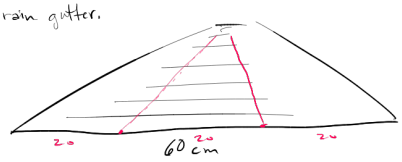
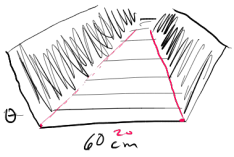
Let's build a rain gutter.

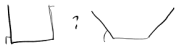


Bend

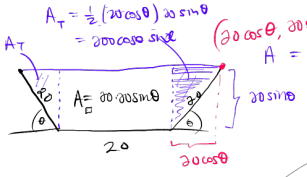


Vertical slice

What degree (θ) of bend will hold the most water.



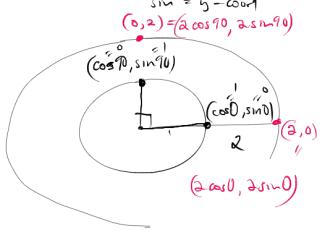
$A_T = \frac{1}{2}(20\cos\theta)\,20\sin\theta$
$= 200\cos\theta\sin\theta$



$(20\cos\theta, 20\sin\theta)$

A = cross-sectional area

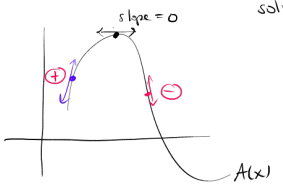
cos = x-coord
sin = y-coord



bigger circle = multiply cosθ, sinθ by length/radius

$$A = 400\cos\theta\sin\theta + 400\sin\theta$$

Calculus Strategy: derivative of $A = A'$
solve $A' = 0$
= slope of A's graph



product rule
$(f\cdot g)' = f\cdot g' + f'\cdot g$
$(\cos\theta)' = -\sin\theta$
$(\sin\theta)' = \cos\theta$

$\cos^2\theta + \sin^2\theta = 1$
$\sin^2\theta = 1 - \cos^2\theta$

$$A = 400\cos\theta\sin\theta + 400\sin\theta$$
$$A = 400(\cos\theta\sin\theta + \sin\theta)$$
$$A' = 400(\cos\theta\cos\theta + (-\sin\theta)(\sin\theta) + \cos\theta)$$
$$= 400(\cos^2\theta - \sin^2\theta + \cos\theta)$$
$$= 400(\cos^2\theta - 1 + \cos^2\theta + \cos\theta)$$
$$= 400(2\cos^2\theta + \cos\theta - 1) = 0$$

divide by 400

$$2\cos^2\theta + \cos\theta - 1 = 0$$

Solve by a quadratic formula trick.

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x = \cos\theta$
$c = -1$
$a = 2$
$b = 1$

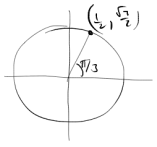
$$\cos\theta = \frac{-1 \pm \sqrt{1 - 4(2)(-1)}}{2\cdot 2} = \frac{-1 \pm \sqrt{9}}{4} = \frac{-1 \pm 3}{4}$$

$$\cos\theta = \frac{-1+3}{4} \quad \text{or} \quad \cos\theta = \frac{-1-3}{4}$$

$\cos\theta = \frac{1}{2}$

$\cos\theta = -1$

$\cos^{-1}(-1) = \pi$
$\Rightarrow$ no area



$\cos^{-1}(\frac{1}{2}) \Rightarrow \theta = \frac{\pi}{3} = 60°$

Differentiation - Product Rule

Find $f'(x)$.

$f = g \cdot h$, $f' = g \cdot h' + g' h$

1. $f(x) = x^2 \sin x$

$f'(x) = 2x \sin x + x^2 \cos x$

2. $f(x) = x^3 \cos x$

$f'(x) = 3x^2 \cos x - x^3 (\sin x)$

$(\tan x)' = \sec^2 x$

3. $f(x) = x \tan x$, $f'(x) = 1 \cdot \tan x + x \cdot \sec^2 x$

4. $f(x) = x^4 \sec x$

5. $f(x) = x^5 e^x$