Wed – week 5

today: Powers, sums, constants & derivatives!

Reminder:

Position	Velocity	Acceleration	Jerk
	rate of change of position	rate of change of velocity	rate of change of acceleration

$s'''(t)$

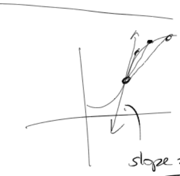
$s(t)$ functions describe position

$s'(t)$ first derivative of position

$s''(t)$ second derivative of position, first derivative of velocity

Formal Def'n of the derivative

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$



slope $= f'(x)$

slope of the derivative.

$f(x) = x^2$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$f'(x) = 2\cdot x^{2-1} = 2x \quad = \lim_{h\to 0} \frac{2xh + h^2}{h} = \lim_{h\to 0} 2x + h = 2x$$

direct sub.

$f(x) = x^3$

$f'(x) \overset{?}{=} 3x^2$

pattern continues –

$$b/c: f'(x) = \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h\to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$

$$= \lim_{h\to 0} 3x^2 + 3xh + h^2 = 3x^2$$

Power Rule

$$f(x) = x^n \qquad f'(x) = n\,x^{n-1}$$

n could be: negative, fraction, any real #

the exponent: bring it down in front, and decrease it by one

Ex.

$f(x) = x^{-3}$ $\qquad f'(x) = -3x^{-4} = \frac{-3}{x^4}$

$f(x) = \frac{1}{x^2} = x^{-2}$ $\qquad f'(x) = -2x^{-3} = \frac{-2}{x^3}$

$f(x) = \sqrt{x} = x^{1/2}$ $\qquad f'(x) = \frac{1}{2}x^{\frac{1}{2}-\frac{2}{2}} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$f(x) = x^{\pi}$ $\qquad f'(x) = \pi x^{\pi - 1}$

$f(x) = 1 = 1\cdot x^0 = x^0$ $\qquad f'(x) = 0\,x^{0-1} = 0$

The derivative of any constant is 0 !!! Look at the graph to see why.

Constant Rule

Let $f(x)$ be given, $k(x) = 18 \cdot f(x)$

$$k'(x) = \lim_{h \to 0} \frac{k(x+h) - k(x)}{h} = \lim_{h \to 0} \frac{18 f(x+h) - 18 \cdot f(x)}{h}$$

$$\lim_{h \to 0} 18 \cdot \frac{f(x+h) - f(x)}{h}$$

$$= 18 \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 18 \cdot f'(x)$$

The derivative passes over constants.

Ex. $f(x) = 5 x^2$ $\quad f'(x) = 5 \cdot 2x = 10x$

the 5 comes along for the ride.

Ex. $f(x) = x^2 + x^3 + 3x^4$

$f'(x) = 2x + 3x^2 + 12x^3$

You can take the derivative of each term.

One more "trick" —

$f(x) = x\sqrt{x}$

$f(x) = x^1 x^{1/2} = x^{3/2}$, $f'(x) = \frac{3}{2} x^{3/2 - 2/2}$

$$= \frac{3}{2} x^{1/2} = \frac{3\sqrt{x}}{2}$$