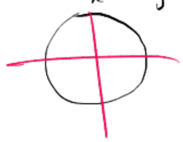
$$\frac{Hi}{Ho} \longrightarrow \frac{Ho\, dHi - Hi\, dHo}{Ho\, Ho}$$

$x^2 + y^2 = 1$



## Differentiation - Quotient Rule

Find $f'(x)$.

1. $f(x) = \tan x = \dfrac{\sin x}{\cos x}$

$$\frac{\cos(x)\cdot(\cos(x)) + \sin(x)(\sin(x))}{\cos^2 x}$$

$\cos(x)$ = x- coord of pt. on unit circle (angle)

$\sin(x)$ = y- cood

$$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)} = \sec^2(x)$$

2. $f(x) = \dfrac{x+1}{x+2}$

$$\frac{(x+2)\cdot 1 - (x+1)\cdot 1}{(x+2)^2} = \frac{x+2-x-1}{(x+2)^2} = \frac{1}{(x+2)^2}$$

3. $f(x) = \dfrac{x^2}{\sin x}$

$$\frac{(\sin x)2x - x^2(\cos(x))}{\sin^2(x)}$$

4. $f(x) = \dfrac{x^3}{\cos x}$

$$\frac{(\cos x)3x^2 - x^3(-\sin(x))}{\cos^2(x)} = \frac{x^2(3\cos(x) + x\sin(x))}{\cos^2(x)}$$

5. $f(x) = \dfrac{\ln x}{x^4}$

$$(\ln(x))' = \frac{1}{x}$$

$$f'(x) = \frac{x^4\cdot\left(\frac{1}{x}\right) - \ln(x)\cdot 4x^3}{x^8} = \frac{x^3 - \ln x\, 4x^3}{x^8}$$

$$= \frac{x^3(1 - 4\ln x)}{x^8}$$

$$\boxed{\frac{1 - 4\ln(x)}{x^5}}$$

6. $f(x) = \sec x = \dfrac{1}{\cos x}$

$$\frac{\cos(x)\cdot 0 - 1\cdot(-\sin(x))}{\cos^2 x} = \frac{\sin(x)}{\cos^2 x}$$

$$= \frac{\sin(x)}{\cos(x)\cdot\cos(x)} = \frac{\sin(x)}{\cos(x)}\cdot\frac{1}{\cos(x)} = \tan x \sec x$$

$$= \sec x \tan x$$

7. $f(x) = \dfrac{e^x + 1}{e^x - 1}$

$$\frac{(e^x-1)(e^x) - (e^x+1)e^x}{(e^x-1)^2} = \frac{e^x(e^x - 1 - e^x - 1)}{(e^x-1)^2} = \frac{-2e^x}{(e^x-1)^2}$$

8. $f(x) = \dfrac{\ln x}{x}$

$$\frac{x\cdot\left(\frac{1}{x}\right) - \ln(x)\cdot 1}{x^2} = \boxed{\frac{1-\ln(x)}{x^2}}$$

9. $f(x) = \dfrac{e^x}{x^3}$

$$\frac{x^3 e^x - e^x\cdot 3x^2}{x^6} = \frac{e^x x^2(x-3)}{x^6}$$

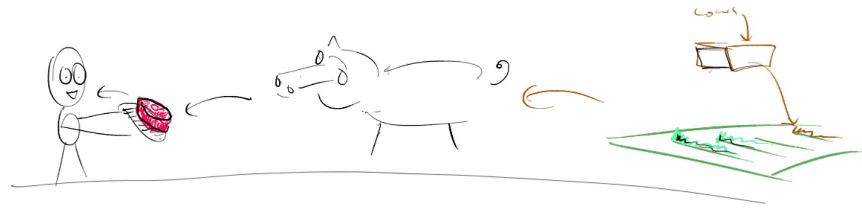
$$= \frac{e^x(x-3)}{x^4}$$

10. $f(x) = \cot x = \dfrac{\cos x}{\sin x}$

$$\frac{\sin(x)(-\sin(x)) - \cos(x)\cos(x)}{\sin^2 x} = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$= \frac{-1}{\sin^2 x} = \boxed{-\csc^2 x}$$

Chain Rule -



$F(x) =$ quality of our health as a function of food $x$.

$C(x) =$ quality of our health as a function of food $x$.

$F(C(x)) =$ quality of our health as function of food the cow eats.

Chain

How does $F$ change with respect to $x$.

$$\underbrace{F'(C(x))}_{\text{How our health changes wrt to cow}} \cdot \underbrace{C'(x)}_{\text{How cow's health changes wrt its food}}$$

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Ex. $f(x) = x^2$

$g(x) = 3x + 1$

the square = "outside" function

$h(x) = f(g(x)) = (3x+1)^2$ inside

$h'(x) = 2(3x+1)^1 \cdot 3$

take the derivative of the outside, copy the inside, take derivative of what you just copied

Ex. $f(x) = (4x^2 + \ln(x) + 5)^3$

$f'(x) = 3(4x^2 + \ln(x) + 5)^2 \cdot \left(8x + \frac{1}{x}\right)$

Ex. $f(x) = \cos((x^2+1)^4)$

$f'(x) = -\sin((x^2+1)^4) \cdot 4(x^2+1)^3 \cdot 2x$

Ex. $x^{14} \xrightarrow{\frac{d}{dx}} 14x^{13} \cdot 1$

1. $$(e^{5x})' = e^{5x} \cdot 5$$

Rule:

u = can be anything

$$(e^{u})' = e^{u} \cdot du$$

$$e^{x} \longrightarrow e^{x} \cdot 1$$

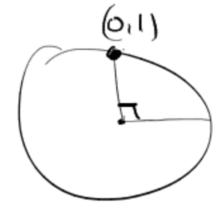
$$e^{2x} \longrightarrow e^{2x} \cdot 2$$

$$e^{3x} \longrightarrow e^{3x} \cdot 3 = 3e^{3x}$$

$$\cos(2x) = -\sin(2x) \cdot 2$$

$$(\cos(u))' = -\sin(u) \cdot du$$

$$\frac{1}{\sin(x)} \neq \sin^{-1}(x) \neq (\sin(x))^{-1}$$



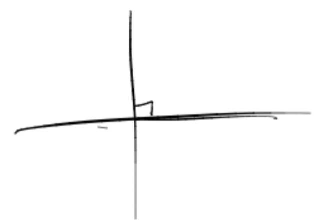

angle required to have height $x$

$$\sin^{-1}(1) = 90^\circ$$

$\tan^{-1}(x)$ = angle whose slope = $x$.

$$\tan x = \frac{\sin x}{\cos x} = \frac{\text{rise}}{\text{run}}$$

$$\tan(90) = \infty$$



$\lim_{x \to \infty} \tan^{-1}(x)$ = angle whose slope approaches $\infty$

$\cancel{90}$ $\pi/2$