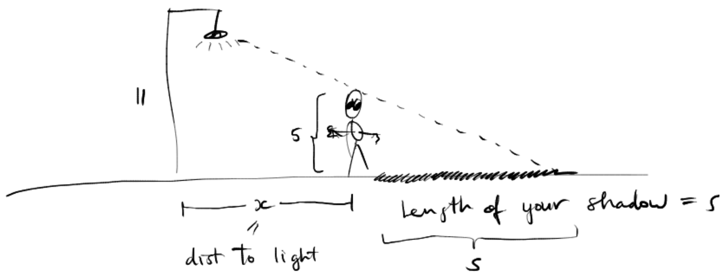
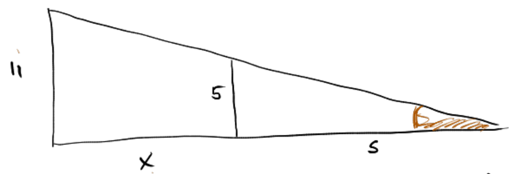
preview what's next



two similar triangles:



Always true for similar triangles

$$\frac{11}{x+s} = \frac{5}{s}$$

If Greg walks at $3\,\frac{m}{h}$ how fast is his shadow moving when he's 20' away from lamp?

$x =$ Greg's dist. to lamp

$\frac{dx}{dt} =$ velocity of Greg : his speed $= 3$

what is $\frac{ds}{dt}$?
$= s'$

$s'(20)$. ← we want to know

$$\frac{11}{x+s} = \frac{5}{s} \Rightarrow 11s = 5x + 5s$$

$$6s = 5x$$

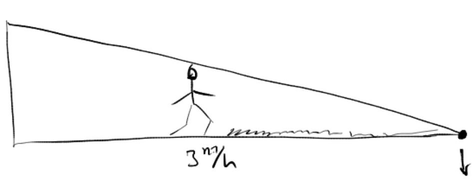
$$s = \frac{5}{6}x$$

take derivative with respect to t.

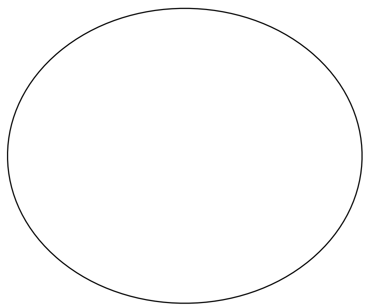
$$\frac{ds}{dt} = \frac{d}{dt}\left(\frac{5}{6}x\right) = \frac{5}{6}\cdot\frac{dx}{dt}$$

now plug in $\frac{dx}{dt} = 3$ so $\frac{ds}{dt} = \frac{5}{6}\cdot 3 = \frac{15}{6} = 2.5\ \frac{m}{h}$

so the length of shadow grows at this rate



↓ Tip of shadow moves @ $5.5\ \frac{m}{h}$



If the radius of a circle increases at 5 ft/sec. How fast is the area increasing when the radius is 3 feet?

Area of a circle?

$$A = \pi r^2$$

r is changing over time.

$$r = r(t)$$

$$\frac{d}{dt}(A) = \frac{d}{dt}(\pi r^2) = \pi \frac{d(r^2)}{dt}$$

$$\frac{dA}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt}$$

when $r = 3$

$$\frac{dr}{dt} = 5$$

so

$$\frac{dA}{dt} = \pi \cdot 2(3) \cdot 5$$

$= 30\pi$ square feet

$\Rightarrow$ area is changing at rate of $30\pi \frac{ft^2}{sec}$

when $r = 10 \Rightarrow \frac{dA}{dt} = \pi \cdot 2(10) \cdot 5 = 100\pi \frac{ft^2}{sec}$

$r = 20 \Rightarrow \frac{dA}{dt} = \pi \cdot 2(20) \cdot 5 = 200\pi \frac{ft^2}{sec}$

$$A = \pi r^2$$

$$C = 2\pi r$$

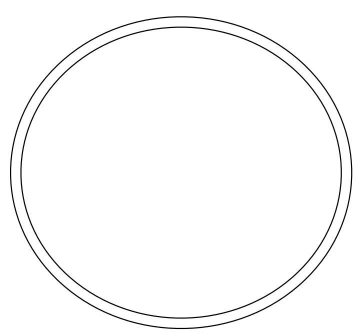
$$\frac{dA}{dr} = 2\pi r$$

$\shortparallel$

rate of change of area.

derivative = instantaneous R of C.

so $\Delta r \to 0$



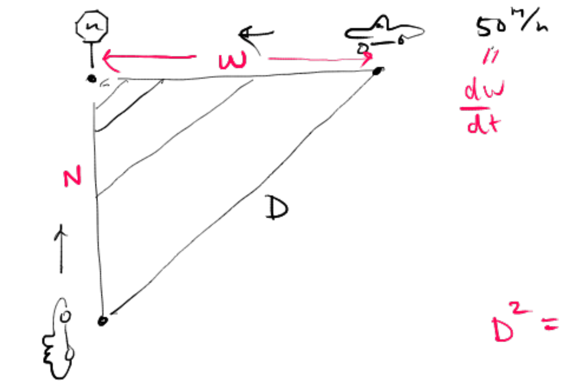
how much new area is there when little circle grows into big?

Circumference $\times$ $\underbrace{\Delta r}_{\text{small change in } r.}$

Volume of Sphere

$\frac{4}{3}\pi r^3$ $\xrightarrow{d/r}$ $4\pi r^2$

Surface area of Sphere.



how fast is D = dist b/w them changing.

we want $\frac{dD}{dt}$

$$D^2 = w^2 + N^2$$

take $\frac{d}{dt}$ of both sides

chain rule b/c each quantity is a functn of time

$$2D \cdot \frac{dD}{dt} = 2w \underbrace{\frac{dw}{dt}}_{=50} + 2N \underbrace{\frac{dN}{dt}}_{=40}$$

when $w = 10$, $N = 30$

$D^2 = 100 + 900 = 1000$

$D = \sqrt{1000} = \sqrt{100} \cdot \sqrt{10}$

$= 10\sqrt{10}$

≈ 31

$$\frac{dD}{dt} = \frac{1}{D}\left(50w + 40N\right)$$

$$\frac{dD}{dt} = \frac{1}{31}\left(500 + 1200\right) = \frac{1700}{31} = 54\ \frac{m}{h}$$

when $w = 1$, $N = 3$

$D^2 = 1^2 + 3^2 = 10$

$D = 3.1$

Now

$$\frac{dD}{dt} = \frac{1}{3.1}\left(50 + 120\right) = \frac{170}{3.1} = 54\ \frac{m}{h}$$

$$\Rightarrow \frac{dD}{dt} = \text{constant}\ 54\ \frac{m}{h}$$