Exam 2 Guide Part A :: Math 271 :: February 27, 2017

Major Concepts

1. Derivatives: definition, concept, applications
2. Derivative computations: applying rules - sum,power, product, quotient, chain
3. Derivative of functions: polys, rational, radicals, trig, exponential, logarithmic

uses power rule

1. (a) Use the definition of the derivative to compute the derivative of $f(x) = \frac{1}{x}$ $= x^{-1} \to -x^{-2} = \frac{-1}{x^2}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\left(\frac{x}{x}\right)\frac{1}{x+h} - \frac{1}{x}\left(\frac{x+h}{x+h}\right)}{h} = \lim_{h\to 0} \frac{\frac{x-(x+h)}{x(x+h)}}{h} = \lim_{h\to 0} \frac{-h}{x(x+h)} \cdot \frac{1}{h} = \lim_{h\to 0} \frac{-1}{x(x+h)}$$

$$= \frac{-1}{x(x+0)} = \boxed{-\frac{1}{x^2}}$$

(b) Use the definition of the derivative to compute the derivative of $f(x) = 3x^2$.

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\lim_{h\to 0} \frac{3(x+h)^2 - 3x^2}{h}$$

$$= \lim_{h\to 0} \frac{3(x^2+2xh+h^2) - 3x^2}{h}$$

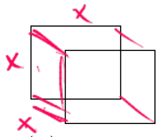
$$\to \lim_{h\to 0} \frac{3x^2 + 6xh + 3h^2 - 3x^2}{h}$$

$$= \lim_{h\to 0} \frac{6xh+3h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(6x+3h)}{\cancel{h}}$$

$$= \lim_{h\to 0} 6x + 3h = 6x$$

2. Find the slope of the tangent line to the curve $y = 3x^2 + 2x + 1$ at $x = 5$.

3. (a) Express the volume V of a cube as a function of its side length x.



$V = x^3$

(b) Take the derivative of your equation. This is the rate of change of the volume of a cube as a function of x.

$V' = 3x^2$

(c) Compute $V'(1)$, $V'(2)$, $V'(5)$ and $V'(10)$.

$= 3 \quad = 12 \quad = 75 \quad = 300$

when the side length of the cube = 10 a tiny change in length $(x + \Delta x)$ produces a change in volume by 300 cube units

(d) Which has the greater affect on volume, lengthening the side length of a *small* cube by 1 or a large cube by by 1?

the derivative of the volume function is much bigger when the side length is large

4. Compute the derivatives of the following.

(a) $3\sin(4t)$

$3\sin(u) \longrightarrow 3\cos(4t)\cdot 4 = 12\cos(4t)$

$\downarrow\ 3\cos(u)\cdot du$

(b) e^{7t}

$e^{7t}\cdot 7 = 7e^{7t}$

$\frac{1}{3} - \frac{3}{3} = -\frac{2}{3}$

(c) $\dfrac{\sqrt[3]{x} - \ln(x)}{\sqrt{x^2+1}}$

$x^{1/3}$

quotient rule:

d Hi, Hi, d Ho

$$\frac{\left(\sqrt{x^2+1}\right)\left(\frac{1}{3}x^{-2/3} - \frac{1}{x}\right) - \left(\sqrt[3]{x} - \ln x\right)\frac{1}{2}\left(x^2+1\right)^{-\frac{1}{2}}\cdot 2x}{\left(x^2+1\right)}$$

$\sqrt{x^2+1} = (x^2+1)^{\frac{1}{2}} \longrightarrow$

$$\frac{6x}{x} + \frac{10}{x} = 6 + \frac{10}{x} \qquad 10x^{-1}$$

5. Suppose that $f(x) = \dfrac{6x+10}{x}$. Evaluate $f'(x)$ and $f'(3)$.

$$-10x^{-2} = \frac{-10}{x^2}$$

$$\frac{x \cdot 6 - (6x+10)}{x^2} = \frac{6x - 6x - 10}{x^2} = \frac{-10}{x^2}$$

$$f'(3) = \frac{-10}{9}$$

6. Suppose that $f(x) = 2x^{-4} + 3x^{-2}$. Evaluate $f'(x)$ and $f'(2)$.

$$-8x^{-5} - 6x^{-3}$$

$$-8(2)^{-5} - 6(2)^{-3}$$

$$\frac{-8}{32} - \frac{6 \cdot 4}{8 \cdot 4} = \frac{-32}{32}$$

$$= -1$$

7. Suppose that $f(x) = \tan \dfrac{1}{x} + 3\cos(x^{-2})$. Evaluate $f'(x)$.

$$\sec^2\left(\frac{1}{x}\right) \cdot \frac{-1}{x^2} - 3\sin(x^{-2})(-2x^{-3})$$

$$\frac{-\sec^2(\frac{1}{x})}{x^2} + \frac{6\sin(\frac{1}{x^2})}{x^3}$$

8. Suppose that $f(x) = \sqrt{x}\sin(x)$.Evaluate $f'(x)$ and $f'(\pi)$.

$$\left(\frac{1}{2}x^{-\frac{1}{2}}\right)\sin(x) + \sqrt{x}\cos(x)$$

$$0 = \frac{\sin(\pi)}{2\sqrt{\pi}} + \sqrt{\pi} \cdot \cos(\pi) = \boxed{-\sqrt{\pi}}$$

$$f'(2) = \frac{(2+\sin 2)(6-\sin(2)) - (6\cdot 2+\cos(2))(1+\cos(2))}{(2+\sin 2)^2}$$

9. Suppose that $f(x) = \dfrac{6x+\cos x}{x+\sin x}$. Evaluate $f'(x)$ and $f'(3)$.

$$f'(x) = \frac{(x+\sin x)(6-\sin x) - (6x+\cos x)(1+\cos x)}{(x+\sin x)^2}$$

(a) Find the slope of the tangent line to $f(x)$ at $x = 2$.

$m = f'(2) = 3.$ (rounding)

(b) Find the instantaneous rate of change of $f(x)$ at $x = 2$.

same!

(c) Find the equation of the tangent line to $f(x)$ at $x = 2$.

$$y = \frac{6.2 + \cos(2)}{2+\sin(2)} = 4$$

$y - y_1 = m(x - x_1)$

roughly

$(x,y) = (2,4)$

$y - 4 = 3(x-2)$

$y = 3x - 2$

10. A bungee jumper's height in feet above the river is given by $f(t) = 876e^{-.17x}\cos(-.05x)$ where t is the number of seconds after jumping. Compute the velocity of the jumper at the following times: $t = 1, t = 19, t = 60$.

"derivative of position"

$f(t) = \underbrace{876e^{-.17x}}\underbrace{\cos(-.05x)} =$ position

chain rule

$$f'(t) = \underbrace{876e^{-.17x}\cdot(-.17)}_{d\ 1^{st}}\underbrace{\cos(-.05x)}_{2^{nd}} - \underbrace{876e^{-.17x}}_{1^{st}}\cdot\underbrace{\sin(-.05x)\cdot(-.05)}_{d\ 2^{nd}}$$

$f'(1)$ $f'(19)$ $f'(60)$

Exam 2 Guide - Derivatives

Find $f'(x)$.

1.$f(x) = e^2$

0

2.$f(x) = -\pi x^3$

$\to -3\pi x^2$

$\to -\pi(3x^2) + 0 \cdot x^3$

$= 0$

$-3.14x^3$

$\downarrow$

$-9.42x^2$

3.$f(x) = x^2 \tan x$

$2x \tan x + x^2 \sec^2 x$

~~4.$f(x) = 4e^{-x} + \sec x - 9 \ln x$~~

$x^7 \longrightarrow 7x^6$

5.$f(x) = (2x^4 - 3e^{2x} + \sin x)^7$

recognize what rule to apply 1st

$7(2x^4 - 3e^{2x} + \sin x)^6 \cdot (8x^3 - 6e^{2x} + \cos x)$

6. $f(x) = \frac{5}{\sqrt{x}} = 5x^{-1/2} = (-\frac{1}{2})5x^{-\frac{3}{2}} = \frac{-5}{2x^{3/2}}$

power rule

$= \frac{-2.5}{x^{3/2}}$

7. $f(x) = (\ln x)^2$

$(\text{inside})^2 \longrightarrow 2(\text{inside})^1 \cdot (\text{derivative of inside})$

$2(\ln x)\cdot \frac{1}{x} = \frac{2\ln x}{x}$

8. $f(x) = \cancel{\sec^3 x}$ $\quad \sin^3 x \longrightarrow \boxed{3\sin^2 x \cdot \cos x}$

$\parallel$

$(\sin x^{\;3}$

$\parallel$

$(\text{inside})^3 \longrightarrow 3(\text{inside})$ d of inside

9. $f(x) = \cancel{e^{\sec^3 x}}$

10. $f(x) = \ln(\sec^3 x)$

11. $f(x) =$ [redacted]

12. $f(x) =$ [redacted]

$x \cdot \sin(x^2)$

product rule
& chain rule

$1 \cdot \sin(x^2) + x \cdot \cos(x^2) \cdot 2x$

13. $f(x) =$ [redacted]

14. $f(x) = \dfrac{e^x - 1}{e^x + 1}$

$$\frac{(e^x+1)e^x - (e^x-1)e^x}{(e^x+1)^2} = \frac{e^{2x} + e^x - e^{2x} + e^x}{(e^x+1)^2}$$

$$= \frac{2e^x}{(e^x+1)^2}$$

15. $f(x) = \dfrac{x^3}{\cos x}$

$$\boxed{\frac{\cos x \cdot 3x^2 + \sin x \cdot x^3}{\cos^2 x}}$$

16. $f(x) =$ [redacted]

17. $f(x) =$ [redacted]

18. $f(x) = \sqrt{x^2 - 1}$