$6 = 6x^0 \xrightarrow{\text{a.d.}} \frac{6x^1}{1} = 6x$

10/12/16
Name

Antiderivatives 1

For problems 1 - 7, find the antiderivative. Check your answer.

1. $\int \frac{1}{2}x^2 - 2x + 6\, dx =$ $\frac{\frac{1}{2}x^3}{3} - \frac{2x^2}{2} + 6x + C$

2. $\int \underbrace{x(2-x)^2}\, dx =$

$\int x(4 - 4x + x^2)\,dx = \int 4x - 4x^2 + x^3\, dx = \boxed{2x^2 - \frac{4x^3}{3} + \frac{x^4}{4} + C}$

3. $\int \sqrt[4]{x^3} + \sqrt[3]{x^4}\, dx$

$\sqrt[2]{x^1} = x^{\frac{1}{2}}$

$\int x^{3/4} + x^{4/3}\, dx = \frac{x^{7/4}}{\left(\frac{7}{4}\right)} + \frac{x^{7/3}}{\left(\frac{7}{3}\right)} + C = \boxed{\frac{4}{7}x^{7/4} + \frac{3}{7}x^{7/3} + C}$

4. $\int 3e^x + 7\sec^2 x\, dx$ what fcn has its derivative $= \sec^2 x$

$3\underbrace{\int e^x dx} + 7\int \sec^2 x\, dx$

$\boxed{3e^x + 7\tan x + C}$

$\int \frac{d}{dx}(e^x)\, dx = \int e^x\, dx$

cancels

e^x

5. $\int 2\sqrt{x} + 6\cos x\, dx$

$\int 2x^{\frac{1}{2}} + 6\cos x\, dx$

$\boxed{\frac{2}{3}\cdot 2x^{3/2} + 6\sin x + C}$

Trick #1 distribute multiplication / division

6. $\int 4+3(1+x^2) \, dx$

$\int 4+3+3x^2 \, dx = \boxed{4x + 3x + x^3 + C}$

$= \boxed{7x + x^3 + C}$

check!

$(7x + x^3 + C)' = 7 + 3x^2$

$4+3 + 3x^2$

$4 + 3(1+x^2)$

7. $\int 6x+\sin x \, dx$

$\boxed{\frac{6x^2}{2} - \cos x + C}$

For problems 8 - 10, find $f(x)$.

8. $f'(x) = 8x^3+12x+3, f(1) = 6$ $\Rightarrow$ determines C.

$f(x) = \int f'(x)\,dx = \int 8x^3 + 12x + 3x^0 = \frac{8x^4}{4} + \frac{12x^2}{2} + 3x + C$

$f(1) = \frac{2(1)^4}{2} + \frac{6(1)^2}{6} + \frac{3(1)}{3} + C \Rightarrow -5 = C$

$6 = 11 + C$

so

$f(x) = 2x^4 + 6x^2 + 3x - 5$

$f'(x) = 8x^3 + 12x + 3$

9. $f'(x) = (x^2-1)/x, \; f(1) = \frac{1}{2}$

$\boxed{f(x)} = \int f'(x)\,dx$

$= \int \frac{x^2-1}{x}\,dx = \int \frac{x^2}{x} - \frac{1}{x}\,dx = \int x - \frac{1}{x}\,dx = \boxed{\frac{x^2}{2} - \ln|x| + C}$

$f(1) = \frac{1}{2}$

$\frac{1}{2} - \underbrace{\ln|1|}_{=0} + C$

$\frac{1}{2} = \frac{1}{2} + C \Rightarrow C = 0$

$\boxed{\frac{x^2}{2} - \ln|x|}$

10. $f''(x) = 4-6x-40x^3, \; \underbrace{f'(0) = 1}_{\text{use now!}}, \; \underbrace{f(0) = 2}_{\text{use now!}}$

think: f'' = accelleration

goal: find f = position

this problem is what you would do to find the position of an object given a formula of its accelleration

strategy

$f = \int f'$

$f' = \int f'' \leftarrow$ given

$f'(x) = \int 4 - 6x - 40x^3 \, dx = 4x - 3x^2 - \frac{40x^4}{4} + C$

$f'(0) = 4(0) - 3(0)^2 - 10(0)^4 + C \Rightarrow C = 1$ update!

1

$\boxed{f'(x) = 4x - 3x^2 - 10x^4 + 1}$

now repeat

$\boxed{f(x)} = \int f'(x)\,dx = \int 4x - 3x^2 - 10x^4 + 1 \, dx$

$= \boxed{\frac{4x^2}{2} - \frac{3x^3}{3} - \frac{10x^5}{5} + x + C}$

$f(0) = C$

2

$\Rightarrow f(x) = 2x^2 - x^3 - 2x^5 + x + 2$

check $f'(x) = 4x - 3x^2 - 10x^4 + 1$

$f''(x) = 4 - 6x - 40x^3$

The substitution technique for integration - (how well can you recognize patterns?)

$\int x^3\,dx = \frac{x^4}{4} + C$ similarly $\int u^3\,du = \frac{u^4}{4} + C$

so the variable doesn't matter.

This <u>even</u> works when the "variable" is a function.

$$\int (x+1)^3\,dx \overset{\text{sub.}}{=} \int u^3\,du = \frac{u^4}{4} \overset{\text{back sub}}{=} \boxed{\frac{(x+1)^4}{4} + C}$$

can't just kick up / divide

try substitution: set $u =$ "inside of a composite function"

$u = x+1$

$\frac{du}{dx} = 1 \Rightarrow$ multiply by dx $du = dx$

$$\int (2x+1)^4\,dx = \int u^4\left(\tfrac{1}{2}\right)du = \tfrac{1}{2}\int u^4\,du = \left(\tfrac{1}{2}\right)\frac{u^5}{5} + C$$

$u = 2x+1$

$\frac{du}{dx} = 2$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$$\boxed{\frac{1}{10}(2x+1)^5 + C}$$

$u = 2x+1$

$du = 2$

$$\frac{1}{2}\int (2x+1)^4\, 2\,dx = \frac{1}{2}\frac{(2x+1)^5}{5}$$

$$= \int u^4\,du$$

$$\frac{1}{10}(2x+1)^5 + C$$