

What we've been doing.

$$f(x) = x^3 \xrightarrow{\text{derivative}} f'(x) = 3x^2$$

(position) $\xrightarrow{\hspace{2cm}}$ (velocity)

(velocity) $\xrightarrow{\hspace{2cm}}$ acceleration

Now, we go backwards

position $\xleftarrow{\text{anti-derivative}}$ velocity

$\frac{1}{3}x^3$ $\xleftarrow{\hspace{2cm}}$ x^2

$\frac{d}{dx} 3 \cdot \frac{1}{3} x^2 = x^2$ $\left| \begin{array}{l} (\frac{1}{3}x^3 + 27)' = x^2 \\ (\frac{1}{3}x^3 + 227)' = x^2 \end{array} \right.$

$\frac{1}{5}x^5$ $\xleftarrow{\hspace{2cm}}$ x^4

$\frac{d}{dx} 5 \cdot \frac{1}{5} x^4 = x^4$

Rule #1:

an anti-derivative of x^n is $\frac{x^{n+1}}{n+1}$

★ Kick it up by one and divide by it

Notation:

$\int x^4 dx = \frac{x^5}{5} + C$

$\int x^4 dx$ $\xrightarrow{\text{what the variable is}}$ $\frac{x^5}{5}$ $+ C$ $\xrightarrow{\text{represents an unknown constant}}$

the most general anti-derivative of x^4 with respect to x .

"the integral of x^4 with respect to x " \xrightarrow{dx}

THERE ARE MANY DIFFERENT FUNCTIONS WITH THE SAME DERIVATIVE

Ex

$$\int x^3 + 2x^5 \, dx = \frac{x^4}{4} + \frac{2x^6}{6} + C$$

$$= \boxed{\frac{x^4}{4} + \frac{x^6}{3} + C}$$

Fraction Exponent

$$x^{1/2} \xrightarrow{d/dx} \frac{1}{2} x^{-1/2} \quad (\text{power rule})$$

$$x^{1/2} \xrightarrow{\text{anti-deriv.}} \frac{x^{3/2}}{(\frac{3}{2})} = \boxed{\frac{2}{3} x^{3/2} + C}$$

check:

$$\left(\frac{2}{3} x^{3/2} + C \right)' = \frac{2}{3} \cdot \frac{3}{2} x^{1/2} = \boxed{x^{1/2}}$$



$$\int \cos x \, dx = \sin x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \underbrace{x(1+x^3)}_{\text{product!}} \, dx \neq \frac{x^2}{2} \left(x + \frac{x^4}{4} \right)$$

|| after distributing

$$\int x + x^4 \, dx = \frac{x^2}{2} + \frac{x^5}{5} + C$$

$$\int 2 \, dx = 2x + C$$

b/c

$$\left(\left(\frac{x^2}{2} \right) \left(x + \frac{x^4}{4} \right) \right)'$$

$$\left(\frac{x^3}{2} + \frac{x^6}{8} \right)'$$

$$\frac{3x^2}{2} + \frac{6x^5}{8}$$

$$b = b x^0 \xrightarrow{\text{a.d.}} b \frac{x^1}{1} = b x$$

10/12/16
Name

Antiderivatives 1

For problems 1 - 7, find the antiderivative. Check your answer.

$$1. \int \frac{1}{2}x^2 - 2x + 6 \, dx = \frac{\frac{1}{2}x^3}{\frac{3}{3}} - \frac{2x^2}{2} + 6x + C$$

$$2. \int x(2-x)^2 \, dx = \int x(4 - 4x + x^2) \, dx = \int 4x - 4x^2 + x^3 \, dx = 2x^2 - \frac{4x^3}{3} + \frac{x^4}{4} + C$$

$$3. \int \sqrt[4]{x^3} + \sqrt[3]{x^4} \, dx = \int x^{3/4} + x^{4/3} \, dx = \frac{x^{7/4}}{(\frac{7}{4})} + \frac{x^{7/3}}{(\frac{7}{3})} + C = \frac{4}{7}x^{7/4} + \frac{3}{7}x^{7/3} + C$$

$$4. \int 3e^x + 7\sec^2 x \, dx$$

$$5. \int 2\sqrt{x} + 6 \cos x \, dx$$