

10/12/16

Name

Antiderivatives 1

For problems 1 - 7, find the antiderivative. Check your answer.

$$1. \int \frac{1}{2}x^2 - 2x + 6 \, dx = \frac{1}{2} \cdot \frac{x^3}{3} - 2 \cdot \frac{x^2}{2} + 6x + C = \frac{1}{6}x^3 - x^2 + 6x + C$$

Check:

$$\frac{d}{dx} \left[\frac{1}{6}x^3 - x^2 + 6x \right] = \frac{1}{6}(3x^2) - 2x + 6(1) = \frac{1}{2}x^2 - 2x + 6$$

$$2. \int x(2-x)^2 \, dx = \int x(4-4x+x^2) \, dx = \int x^3 - 4x^2 + 4x \, dx$$

$$= \frac{x^4}{4} - 4 \cdot \frac{x^3}{3} + 4 \cdot \frac{x^2}{2} + C = \frac{1}{4}x^4 - \frac{4}{3}x^3 + 2x^2 + C$$

Check:

$$\frac{d}{dx} \left[\frac{1}{4}x^4 - \frac{4}{3}x^3 + 2x^2 \right] = \frac{1}{4}(4x^3) - \frac{4}{3}(3x^2) + 2(2x) = x^3 - 4x^2 + 4x$$

$$3. \int \sqrt[4]{x^3} + \sqrt[3]{x^4} \, dx = \int x^{3/4} + x^{4/3} \, dx = \frac{x^{7/4}}{7/4} + \frac{x^{7/3}}{7/3} + C = \frac{4}{7}x^{7/4} + \frac{3}{7}x^{7/3} + C$$

Check:

$$\frac{d}{dx} \left[\frac{4}{7}x^{7/4} + \frac{3}{7}x^{7/3} \right] = \frac{4}{7} \left(\frac{7}{4}x^{3/4} \right) + \frac{3}{7} \left(\frac{7}{3}x^{4/3} \right) = x^{3/4} + x^{4/3} = \sqrt[4]{x^3} + \sqrt[3]{x^4}$$

$$4. \int 3e^x + 7 \sec^2 x \, dx = \int 3e^x \, dx + \int 7 \sec^2 x \, dx = 3 \int e^x \, dx + 7 \int \sec^2 x \, dx$$

$$= 3e^x + 7 \tan x + C$$

Check:

$$\frac{d}{dx} [3e^x + 7 \tan x] = 3e^x + 7 \sec^2 x$$

$$5. \int 2\sqrt{x} + 6 \cos x \, dx = \int 2x^{1/2} + 6 \cos x \, dx = 2 \cdot \frac{x^{3/2}}{3/2} + 6(\sin x) + C = \frac{4}{3}x^{3/2} + 6 \sin x + C$$

Check:

$$\frac{d}{dx} \left[\frac{4}{3}x^{3/2} + 6 \sin x \right] = \frac{4}{3} \left(\frac{3}{2}x^{1/2} \right) + 6 \cos x = 2x^{1/2} + 6 \cos x = 2\sqrt{x} + 6 \cos x$$

$$6. \int 4 + 3(1 + x^2)^{-1} \, dx = \int 4 + 3 \left(\frac{1}{x^2 + 1} \right) \, dx = 4x + 3 \tan^{-1}(x) + C$$

Check:

$$\frac{d}{dx} [4x + 3 \tan^{-1}(x)] = 4(1) + 3 \left(\frac{1}{x^2 + 1} \right) = 4 + 3(1 + x^2)^{-1}$$

$$7. \int 6x + \sin x \, dx = 6 \cdot \frac{x^2}{2} + (-\cos x) + C = 3x^2 - \cos x + C$$

Check:

$$\frac{d}{dx} [3x^2 - \cos x] = 3(2x) - (-\sin x) = 6x + \sin x$$

For problems 8 - 10, find $f(x)$.

$$8. f'(x) = 8x^3 + 12x + 3, f(1) = 6$$

$$f(x) = \int 8x^3 + 12x + 3 \, dx = 8 \cdot \frac{x^4}{4} + 12 \cdot \frac{x^2}{2} + 3x + C = 2x^4 + 6x^2 + 3x + C$$

$$6 = f(1) = 2(1)^4 + 6(1)^2 + 3(1) + C \longrightarrow C = 6 - 2 - 6 - 3 = -5$$

$$\text{Answer: } f(x) = 2x^4 + 6x^2 + 3x - 5$$

$$9. f'(x) = (x^2 - 1)/x, f(1) = \frac{1}{2}.$$

$$f(x) = \int (x^2 - 1)/x \, dx = \int x - \frac{1}{x} \, dx = \frac{1}{2}x^2 - \ln|x| + C$$

$$\frac{1}{2} = f(1) = \frac{1}{2}(1)^2 - \ln|1| + C \longrightarrow C = 0$$

$$\text{Answer: } f(x) = \frac{1}{2}x^2 - \ln|x|$$

$$10. \ f''(x) = 4 - 6x - 40x^3, \ f'(0) = 1, \ f(0) = 2$$

$$f'(x) = \int 4 - 6x - 40x^3 \, dx = -10x^4 - 3x^2 + 4x + C$$

$$1 = f'(0) = -10(0)^4 - 3(0)^2 + 4(0) + C \longrightarrow C = 1$$

$$f(x) = \int -10x^4 - 3x^2 + 4x + 1 \, dx = -2x^5 - x^3 + 2x^2 + x + C$$

$$2 = f(0) = -2(0)^5 - (0)^3 + 2(0)^2 + (0) + C \longrightarrow C = 2$$

$$\text{Answer: } f(x) = -2x^5 - x^3 + 2x^2 + x + 2$$