10/14/16
Name

Antiderivatives 3

Find the indicated antiderivative. Check your answers.

1. Use $u = \tan^{-1} x \to du = \frac{1}{x^2+1}\, dx \to \left(x^2+1\right)\, du = dx$

$$\int \frac{\left(\tan^{-1} x\right)^3}{x^2+1}\, dx = \int \frac{(u)^3}{x^2+1}\left(x^2+1\right)\, du = \int u^3\, du = \frac{1}{4}u^4 + C = \frac{1}{4}\left(\tan^{-1} x\right)^4 + C$$

Check

$$\frac{d}{dx}\left[\frac{1}{4}\left(\tan^{-1} x\right)^4\right] = \frac{4}{4}(4)\left(\tan^{-1} x\right)^3 \frac{d}{dx}\left[\tan^{-1} x\right] = \left(\tan^{-1} x\right)^3\left(\frac{1}{x^2+1}\right) = \frac{\left(\tan^{-1} x\right)^3}{x^2+1}$$

2. Use $u = x^2+1 \to du = 2x\, dx \to \frac{1}{2x}\, du = dx$

$$\int \frac{4x}{x^2+1}\, dx = \int \frac{4x}{u}\cdot\frac{1}{2x}\, du = \int \frac{2}{u}\, du = 2\int \frac{1}{u}\, du = 2\ln|u| + C = 2\ln\left(x^2+1\right) + C$$

Check:

$$\frac{d}{dx}\left[2\ln\left(x^2+1\right)\right] = 2\left(\frac{\frac{d}{dx}\left[x^2+1\right]}{x^2+1}\right) = \frac{2(2x)}{x^2+1} = \frac{4x}{x^2+1}$$

3. Use $u = x^2+1 \to du = 2x\, dx \to \frac{1}{2x}\, du = dx$

$$\int \frac{4x}{\left(x^2+1\right)^2}\, dx = \int \frac{4x}{(u)^2}\cdot\frac{1}{2x}\, du = \int \frac{2}{u^2}\, du = \int 2u^{-2}\, du = 2\frac{u^{-1}}{-1} + C = -\frac{2}{x^2+1} + C$$

Check:

$$\frac{d}{dx}\left[-\frac{2}{x^2+1}\right] = \frac{d}{dx}\left[-2\left(x^2+1\right)^{-1}\right] = -2(-1)\left(x^2+1\right)^{-2}\frac{d}{dx}\left[x^2+1\right] = \frac{2(2x)}{\left(x^2+1\right)^2}$$

4.

$$\int \frac{5}{x^2+1}\, dx = 5\int \frac{1}{x^2+1}\, dx = 5\tan^{-1} x + C$$

Check:

$$\frac{d}{dx}\left[5\tan^{-1} x\right] = 5\cdot\frac{1}{x^2+1} = \frac{5}{x^2+1}$$

5. Use #2 and #4 above.

$$\int \frac{4x+5}{x^2+1}\, dx = \int \frac{4x}{x^2+1} + \frac{5}{x^2+1}\, dx = 2\ln\left(x^2+1\right) + 5\tan^{-1} x + C$$

6. Use $u = 1 - x^2 \to du = -2x\,dx \to -\frac{1}{2x}\,du = dx$

$$\int x\sqrt{1-x^2}\,dx = \int x\sqrt{u}\left(-\frac{1}{2x}\right)\,du = \int -\frac{1}{2}u^{1/2}\,du = -\frac{1}{2}\cdot\frac{u^{3/2}}{3/2} + C$$

$$= -\frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C = -\frac{1}{3}\left(1-x^2\right)^{3/2} + C$$

Check:

$$\frac{d}{dx}\left[-\frac{1}{3}\left(1-x^2\right)^{3/2}\right] = -\frac{1}{3}\cdot\frac{3}{2}\left(\right)^{1/2}\frac{d}{dx}\left[1-x^2\right] = -\frac{1}{2}\sqrt{1-x^2}(-2x) = x\sqrt{1-x^2}$$

7. Use $u = 1 - x^2 \to du = -2x\,dx \to -\frac{1}{2x}\,du = dx$

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = int\frac{x}{\sqrt{u}}\left(-\frac{1}{2x}\right)\,du = \int -\frac{1}{2}u^{-1/2}\,du = -\frac{1}{2}\cdot\frac{u^{1/2}}{1/2} + C$$

$$= -\frac{1}{2}\cdot\frac{2}{1}\sqrt{1-x^2} + C = -\sqrt{1-x^2} + C$$

Check:

$$\frac{d}{dx}\left[-\sqrt{1-x^2}\right] = -\left(\frac{1}{2}\right)\left(1-x^2\right)^{-1/2}\frac{d}{dx}\left[1-x^2\right] = -\frac{1}{2}\cdot\frac{1}{\sqrt{1-x^2}}(-2x) = \frac{x}{\sqrt{1-x^2}}$$

8. Use $u = \sin^{-1}x \to du = \frac{1}{\sqrt{1-x^2}}\,dx \to \sqrt{1-x^2}\,du = dx$

$$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \int \frac{u}{\sqrt{1-x^2}}\left(\sqrt{1-x^2}\right)\,du = \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}\left(\sin^{-1}x\right)^2 + C$$

Check:

$$\frac{d}{dx}\left[\frac{1}{2}\left(\sin^{-1}x\right)^2\right] = \frac{1}{2}(2)\left(\sin^{-1}x\right)^1\frac{d}{dx}\left[\sin^{-1}x\right] = \sin^{-1}x\left(\frac{1}{\sqrt{1-x^2}}\right) = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$$

9.

$$\int \frac{3}{\sqrt{1-x^2}}\,dx = 3\int \frac{1}{\sqrt{1-x^2}}\,dx = 3\sin^{-1}x + C$$

Check:

$$\frac{d}{dx}\left[3\sin^{-1}x\right] = 3\left(\frac{1}{\sqrt{1-x^2}}\right) = \frac{3}{\sqrt{1-x^2}}$$

10. Use #7 and #9

$$\int \frac{x-3}{\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}} - \frac{3}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2} - 3\sin^{-1}x + C$$