10/18/16
Name

Antiderivatives 5

Find the indicated antiderivative. Check your answers.

1. Use $u = x^2 + 1 \to du = 2x\,dx \to \frac{1}{2x}\,du = dx$

$$\int \frac{2x}{x^2+1}\,dx = \int \frac{2x}{u} \cdot \frac{1}{2x}\,du = \int \frac{1}{u}\,du = \ln|u| + C = \ln\left(x^2+1\right) + C$$

Check:

$$\frac{d}{dx}\left[\ln\left(x^2+1\right)\right] = \frac{\frac{d}{dx}\left[x^2+1\right]}{x^2+1} = \frac{2x}{x^2+1}$$

2. Use $u = 1 + \sin x \to du = \cos x\,dx \to \frac{1}{\cos x}\,du = dx$

$$\int \frac{\cos x}{1+\sin x}\,dx = \frac{\cos x}{u} \cdot \frac{1}{\cos x}\,du = \int \frac{1}{u}\,du = \ln|u| + C = \ln|1+\sin x| + C$$

Check:

$$\frac{d}{dx}\left[\ln|1+\sin x|\right] = \frac{\frac{d}{dx}\left[1+\sin x\right]}{1+\sin x} = \frac{\cos x}{1+\sin x}$$

3. Use $u = 1 + x^4 \to du = 4x^3\,dx \to \frac{1}{4x^3}\,du = dx$

$$\int \frac{x^3}{1+x^4}\,dx = \int \frac{x^3}{u} \cdot \frac{1}{4x^3}\,du = \int \frac{1}{4}\left(\frac{1}{u}\right)\,du = \frac{1}{4}\ln|u| + C = \frac{1}{4}\ln\left(1+x^4\right) + C$$

Check:

$$\frac{d}{dx}\left[\frac{1}{4}\ln\left(1+x^4\right)\right] = \left(\frac{1}{4}\right)\frac{\frac{d}{dx}\left[1+x^4\right]}{1+x^4} = \frac{1}{4} \cdot \frac{4x^3}{1+x^4} = \frac{x^3}{1+x^4}$$

4. Use $u = e^x + 2x + 7 \to du = (e^x + 2)\ dx \to \frac{1}{d^x+2}\,du = dx$

$$\int \frac{e^x+2}{e^x+2x+7}\,dx = \int \frac{e^x+2}{u} \cdot \frac{1}{e^x+2}\,du = \int \frac{1}{u}\,du = \ln|u| + C = \ln|e^x+2x+7| + C$$

Check:

$$\frac{d}{dx}\left[\ln|e^x+2x+7|\right] = \frac{\frac{d}{dx}\left[e^x+2x+7\right]}{e^x+2x+7} = \frac{e^x+2}{e^x+2x+7}$$

5. Use $u = 2x + 5 \to du = 2\,dx \to \frac{1}{2}\,du = dx$

$$\int \frac{3}{2x+5}\,dx = \int \frac{3}{u} \cdot \frac{1}{2}\,du = \int \frac{3}{2}\left(\frac{1}{u}\right)\,du = \frac{3}{2}\ln|u| + C = \frac{3}{2}\ln|2x+5| + C$$

Check:

$$\frac{d}{dx}\left[\frac{3}{2}\ln|2x+5|\right] = \frac{3}{2} \cdot \frac{\frac{d}{dx}\left[2x+6\right]}{(2x+5)} = \frac{3}{2} \cdot \frac{2}{(2x+5)} = \frac{3}{2x+5}$$

6. Use $u = x^2 + 1 \to du = 2x\,dx \to \frac{1}{2x}\,du = dx$

$$\int \frac{2x}{\left(x^2+1\right)^2}\,dx = \int \frac{2x}{\left(u\right)^2} \cdot \frac{1}{2x}\,du = \int u^{-2}\,du = -u^{-1} + C = -\frac{1}{x^2+1} + C$$

Check:

$$\frac{d}{dx}\left[-\frac{1}{x^2+1}\right] = \frac{d}{dx}\left[-\left(x^2+1\right)^{-1}\right] = -(-1)\left(x^2+1\right)^{-2}(2x) = \frac{2x}{\left(x^2+1\right)^2}$$

7. Use $u = 1 + \sin x \to du = \cos x\,dx \to \frac{1}{\cos x}\,du = dx$

$$\int \frac{\cos x}{\left(1+\sin x\right)^2}\,dx = \int \frac{\cos x}{\left(u\right)^2}\frac{1}{\cos x}\,du = \int u^{-2}\,du = -u^{-1} + C = -\frac{1}{1+\sin x} + C$$

Check:

$$\frac{d}{dx}\left[-\frac{1}{1+\sin x}\right] = \frac{d}{dx}\left[-(1+\sin x)^{-1}\right] = -(-1)(1+\sin x)^{-2}(\cos x) = \frac{\cos x}{(1+\sin x)^2}$$

8. Use $u = 1 + x^4 \to du = 4x^3\,dx \to \frac{1}{4x^3}\,du = dx$

$$\int \frac{x^3}{\left(1+x^4\right)^3}\,dx = \int \frac{x^3}{(u)^3} \cdot \frac{1}{4x^3}\,du = \int \frac{1}{4}u^{-3}\,du = \frac{1}{4}\left(-\frac{1}{2}\right)u^{-2} + C = -\frac{1}{8\left(1+x^4\right)^2} + C$$

Check:

$$\frac{d}{dx}\left[-\frac{1}{8\left(1+x^4\right)^2}\right] = \frac{d}{dx}\left[-\frac{1}{8}\left(1+x^4\right)^{-2}\right] = -\frac{1}{8}(-2)\left(1+x^4\right)^{-3}\left(4x^3\right) = \frac{x^3}{\left(1+x^4\right)^3}$$

9. Use $u = e^x + 2x + 7 \to du = (e^x + 2)\ dx \to \frac{1}{d^x+2}\, du = dx$

$$\int \frac{e^x + 2}{(e^x + 2x + 7)^3}\, dx = \int \frac{(e^x + 2)}{(u)^3} \cdot \frac{1}{(e^x + 2)}\, du = \int u^{-3}\, du$$

$$= -\frac{1}{2}u^{-2} + C = -\frac{1}{2\,(e^x + 2x + 7)^2} + C$$

Check:

$$\frac{d}{dx}\left[-\frac{1}{2\,(e^x + 2x + 7)^2}\right] = \frac{d}{dx}\left[-\frac{1}{2}\,(e^x + 2x + 7)^{-2}\right]$$

$$= -\frac{1}{2}(-2)\,(e^x + 2x + 7)^{-3}\,(e^x + 2) = \frac{e^x + 2}{(e^x + 2x + 7)^3}$$

10. Use $u = 2x + 5 \to du = 2\, dx \to \frac{1}{2}\, du = dx$

$$\int \frac{3}{(2x + 5)^4}\, dx = \int \frac{3}{(u)^4}\left(\frac{1}{2}\right) du = \int \frac{3}{2}u^{-3}\, du = \frac{3}{2}\left(-\frac{1}{3}\right) u^{-2} + C = -\frac{1}{2(2x + 5)^3} + C$$

Check:

$$\frac{d}{dx}\left[-\frac{1}{2(2x + 5)^3}\right] = \frac{d}{dx}\left[-\frac{1}{2}(2x + 5)^{-3}\right] = -\frac{1}{2}(-3)(2x + 5)^{-4}(2) = \frac{3}{(2x + 5)^4}$$