#28 – Ch. 0, Show:

$2^n 3^{2n} - 1 \mod 17 = 0$

Proceed by induction

$n=1$: $2 \cdot 3^2 - 1 = 17$

$n=k$: $2^k 3^{2k} - 1 \mod 17 = 0$, $2^k 3^{2k} - 1 = 17m$

$$\ast 18 \qquad \ast 18$$

$$2 \cdot 3^2 (2^k 3^{2k} - 1) = 18 \cdot 17m$$

$$2^{k+1} 3^{2(k+1)} - 18 = 18 \cdot 17m$$

$$+17 \qquad +17$$

$$2^{k+1} \cdot 3^{2(k+1)} - 1 = 18 \cdot 17m + 17 = 17(18m+1)$$

QED

#38 – Ch. 0:

$\forall n \in \mathbb{Z}$, $n^3 \bmod 6 = n \bmod 6$

1. $n^3 - n = n(n^2-1) = n(n-1)(n+1)$

2. Recall $n^3 \bmod 6 = n \bmod \iff n^3 - n$ is divisible by 6.

3. Exactly one of the following holds: $n = 6k$, $n = 6k+1$, $n = 6k+2$, $n = 6k+3$, $n = 6k-2$, $n = 6k-1$

(i) $n = 6k \Rightarrow n^3 = 6(6^2k^3)$ so $n^3 \bmod 6 = n \bmod 6 = 0$.

(ii) $n = 6k+1 \Rightarrow n^3 - n = (6k+1)\cdot(6k+1-1)(6k+1+1)$ using 1

$= 6k \cdot (6k+1)(6k+2) \Rightarrow n^3 - n \bmod 6 = 0$, so by 2 we're done.

(iii) $n = 6k+2 \Rightarrow n^3 - n \overset{1}{=} (6k+2)(6k+1)(6k+3) = 2(3k+1)(6k+1)(3)(2k+1) = 6k$ for $k \in \mathbb{Z}$.

By 2 we're done.

(iv) $n = 6k+3 \Rightarrow n^3 - n = (6k+3)(6k-2)(6k+4) = 3(k+1)(2)(3k-1)(6k+4) = 6k'$, for $k' \in \mathbb{Z}$.

By 2 we're done.

(v) $n = 6k-2 \Rightarrow n^3 - n = 2(3k-1)\overbrace{(6k-3)}^{3(2k-1)}(6k-1) = 6k''$, for $k'' \in \mathbb{Z}$. By 2 we're done.

(vi) $n = 6k-1 \Rightarrow n^3 - n = (6k-1)(6k-2)(6k) = 6k_4$, for $k_4 \in \mathbb{Z}$. By 2 we're done.

62. Ch. 0

3,5,7 are the only consecutive odd prime integers.

By working examples notice how any group of consecutive odd integers are divisible by 3.

Let $p_1 = 2k+1$ be arbitrary, then $p_2 = 2k+3$, $p_3 = 2k+5$ are (arbitrary) consecutive odd ints

If p_1 is divisible by 3 the theorem is proved.

Assume p_1 is not divisible by 3. There are two cases.

Case 1. $p_1 = 3k+1$, Case 2: $p_1 = 3k+2$.

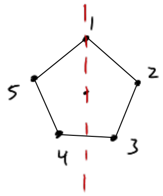
Case 1: $p_2 = p_1 + 2$ by def'n so $p_2 = 3k+3 = 3(k+1)$ so $3 \mid p_2$.

Case 2: $p_3 = p_1 + 4$ by def'n so $p_3 = 3k+2+4 = 3k+6 = 3(k+2)$ so $3 \mid p_3$.

Since this triple was arbitrary, we're done.

H/W

Chapter 1: 4,6,9,10,13,16,19,24

(4.) 

D_5 : 5 Rotations, w/ angles $0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}$

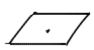
5 reflections, each fixing the center & a unique vertex.

(6.) there are two sides to an n-sided figure, one side we denote +, the other −.
We then say one side has positive orientation, the other negative orientation.
Any rotation preserves the orientation, & any symmetry that preserves orientation is a rotation.
" reflection changes " " " " " changes " " " reflection.
Any reflection followed by a reflection preserves the orientation (+ → − → +) & is thus a reflection.

(9.) See above.

(10.) Odd number of reflections ⇒ reflection.

(13.) $R_{90}V = VR_{270}$

(16)  $G = \{R_0, R_{180}\}$ |  $G = \{R_0, R_{180}, V, H\}$

(19) D_6

(24) $\mathbb{Z}_4$ D_5 D_4 $\mathbb{Z}_2$

D_4 $\mathbb{Z}_3$ D_3 D_{16}

D_7 D_4 D_5 D_{10}