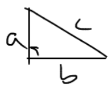
MA541

Text 1.1 – 1.4

Def'n: Differential Geometry: The study (usually) curved spaces using calculus.

But since the derivative is (local), & curved spaces are often modelled well by flat spaces, so tools of Euclidean Geometry are still useful.

Ex! Pythagorean Theorem! (only true in Euclidean Geometry)



$a^2 + b^2 = c^2$

geometry timeline

Calculus

Hyperbolic geom

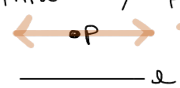
600 BCE — Thales of Miletus

500 BCE — Pythagoras

300 BCE — Euclid (axiomatic geometry) } natural; 1 longer, (Euclid's Parallel Postulate

Thru any pt. P not on line ℓ ∃! line k parallel to ℓ

there exists a unique

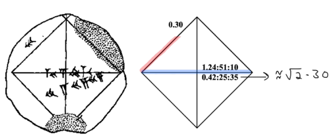


1700's — Lambert, Saccheri, Gauss

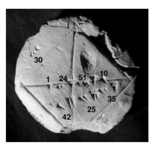
1800's — Lobachevsky, Bolyai

Beltrami

1800 BC



Good Approximation: To $\sqrt{2}$



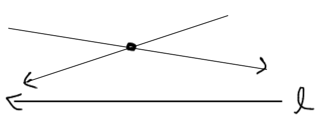
Babylonian Clay Tablet YBC 7289

$1 + \frac{24}{60} + \frac{51}{60^2} + \frac{10}{60^3} \approx \sqrt{2}$

immediatly this postulate seemed diff. – wasn't until 1700s till we realized why.

Lob., & Bolyai (Gauss) discovered a new perfectly valid alternate geometry by: replaced Post 5 w/ this one:

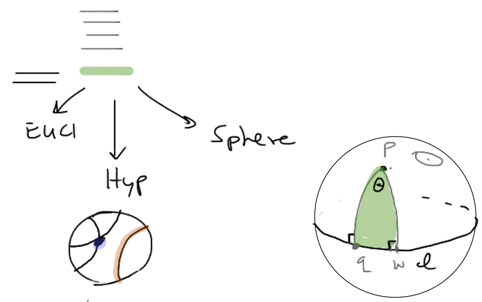
Hyperbolic Axiom: There are at least two parallel lines thru P, disjoint from ℓ.

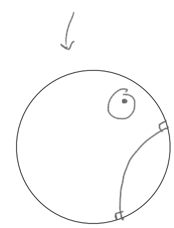


1.3 Angular Excess of a Spherical Δ.

Spherical Axiom: there are NO lines thru P, parallel to ℓ

(lines = great circle)





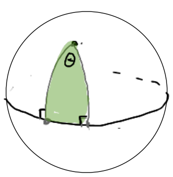
In spherical geometry the angle sum > π.

can you draw a great circle thru P, $\parallel$ to ℓ ?

$\mathcal{E}$ = Angle Sum of Δ $- \pi$

$\mathcal{E} = \theta$

Ex:



So $\mathcal{E} = \frac{\pi}{2} + \frac{\pi}{2} + \theta - \pi = \theta$

Let R = radius,

Total Area of Sphere : $4\pi R^2$

Hemisphere : $2\pi R^2$

What fraction of Hemisphere Area does our triangle have?

$\frac{\theta}{2\pi}$

triangle Area $= A = \frac{\theta}{2\pi} 2\pi R^2 = \theta R^2$

$A = \theta R^2$

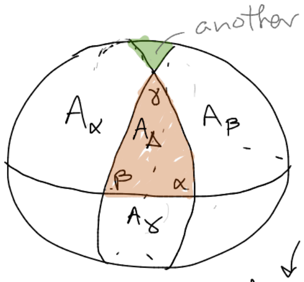
$A = \mathcal{E} R^2$

combine

$$\boxed{\mathcal{E} = \frac{1}{R^2} A}$$

angles area

Harriot's Theorem: this holds for any spherical Δ.



$A_\Delta + A_\alpha = \frac{\alpha}{2\pi} 2\pi R^2 = 2\alpha R^2$

portion of upper hemi det. by α

$A_\Delta + A_\beta = 2\beta R^2$

$A_\Delta + A_\gamma = 2\gamma R^2$

add

$3A_\Delta + A_\alpha + A_\beta + A_\gamma = 2R^2(\alpha + \beta + \gamma)$ (1)

(2) Area of upper hemi $= A_\Delta + A_\alpha + A_\beta + A_\gamma = 2\pi R^2$

(1) − (2) $= 2A_\Delta = 2R^2(\alpha + \beta + \gamma - \pi) = 2R^2 \cdot \mathcal{E}$

$$\boxed{\mathcal{E} = \frac{1}{R^2} A_\Delta}$$