The notes below are a record of what was written in class.

Towards the end of the notes, an error crept in which make what is written difficult to follow.

It will be easiest to read through this document

https://euclid.nmu.edu/~joshthom/teaching/ma541/week13/covar_deriv_notes.pdf

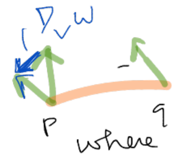
instead of making sense of what is below:

Derivation of Riemann Curvature Tensor

Covariant Deriv:

eval. @ p.

$$\nabla_v w \approx \frac{w_{\parallel}(q \rightsquigarrow p) - w(p)}{\epsilon}$$



where is it evaluated? p? q?

change from w to $w_{\parallel}$

or [from $w_{\parallel}$ to w]

$\updownarrow$

$\nabla_v w(p)$

Vector Holonomy

$$R_u(K) = \delta_K(\angle w_{\parallel})$$

fiducial vector field — edge

measures angle from fixed fiducial (reference) vector field to p.T'd $w_{\parallel}$ along K.

key: when computing holonomy — choose $w = u$ (of unit length) as the vector field

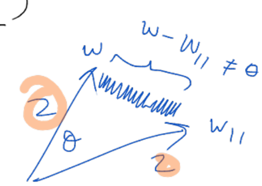
NOW

$$-(\nabla_v(\ ))\approx -\left(\frac{w_{\parallel}(q \rightsquigarrow\ ) - w(\ )}{\epsilon}\right) \approx \frac{w(\ ) - w_{\parallel}(q \rightsquigarrow\ )}{\epsilon}$$

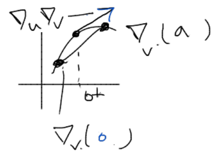
vector field

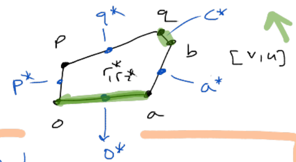
Fiducid vector field

$$\approx R_u(K)$$



By Mean Val. Thm
negative vector holonomy along $K = oa$ is





$$-\delta_{oa}(\langle w_{\|}, w) \approx w(o^*) - w_{\|}(o^*)$$
$$\approx \nabla_v w(o^*)$$

$$\boxed{\frac{\nabla_v(a) - \nabla_v(o)}{\delta u} \overset{M.V.T}{=} \nabla_u(\nabla_v(o^*))}$$

Avg. Rate of Change

So... Computing the full negative holonomy around the loop:
5 segments

$$-R_w(L) \approx -R_w(oa) - R_w(ab) - R_w(bq) - R_w(q,p) - R_w(po)$$

$$\approx \nabla_{\delta u\, u} \overbrace{w(o^*)}^{\text{vector field @ } o^*} + \nabla_{\delta v\, v} w(a^*) + \nabla_{c} w(c^*) - \nabla_{\delta u\, u} w(q^*) - \nabla_{\delta v\, v} w(p^*)$$

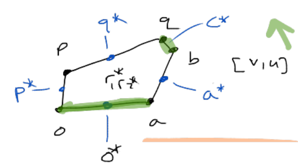
$$c = \delta u \delta v [v,u]$$

$$\approx \delta u \nabla_u w(o^*) - \delta u \nabla_u w(q^*) + \delta v \nabla_v(a^*) - \delta v \nabla_v(p^*) + \delta u \delta v \nabla_{[v,u]}(c^*)$$

$$\delta u(\nabla_u w(o^*) - \nabla_u w(q^*)) \qquad \delta v(\nabla_v(a^*) - \nabla_v(p^*))$$

$$\approx \delta u (-) \nabla_v(\nabla_u r^*) \cdot \delta v \qquad = \delta v(\nabla_u \nabla_v(r^*)(-\delta u)$$

$$\approx -\delta u \delta v \nabla_v \nabla_u(r_1^*) - \delta u \delta v \nabla_u \nabla_v(r_2^*) - \delta u \delta v \nabla_{[u,v]}(c^*)$$



$$\approx \delta u \delta v \left(\underbrace{\nabla_u \nabla_v - \nabla_v \nabla_u}_{[\nabla_v, \nabla_u]} - \nabla_{[u,v]} \right)$$

$$-\underbrace{\left[\delta u \nabla_u w(q^*) - \delta u \nabla_u w(o^*)\right]}_{\nabla_v(\nabla_u(r^*)}$$

$$\approx \delta u \delta v \left([\nabla_v, \nabla_u] - \nabla_{[u,v]} \right)(w)$$

$\div\ \delta u \delta v$ & limit to 0

$$\Rightarrow \quad R(u,v;w) = \lim \frac{-R_w(L)}{\delta u \delta v} = [\nabla_u, \nabla_v] w - \nabla_{[u,v]} w$$