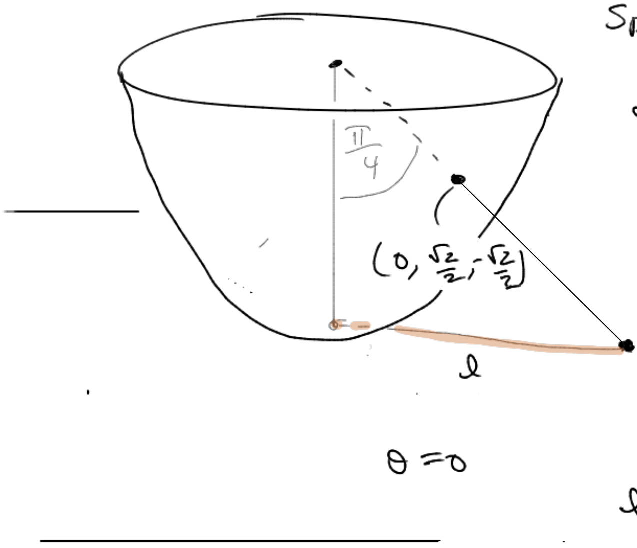



Sphere w/ $R = 1$ under Projection

$$ds^2 = \frac{1}{1+(r/R)^2}\left[\frac{dr^2}{1+(r/R)^2} + r^2 d\theta^2\right]$$

$$= \frac{1}{1+r^2}\left(\frac{dr^2}{1+r^2}\right) = \frac{dr^2}{(1+r^2)^2} \quad (d\theta = 0)$$

$$\Rightarrow \quad ds = \frac{1}{1+r^2}dr$$

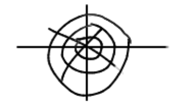
$$\ell = \int_0^1 \frac{1}{1+r^2}dr = \tan^{-1}(r)\Big|_0^1 = \tan^{-1}(1) - \tan(0)$$

$$= \frac{\pi}{4}$$

arc from south pole, up to the indicated point, making angle pi/4 with pole. the arc is an eighth of a circumference.

Ex

Coord Indep of K —



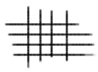
Start w/ Euclidean metric: $ds^2 = dx^2 + dy^2$

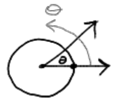
$u \rightsquigarrow u^2 = r$, $v \rightsquigarrow \theta = \theta$ $\}$ $(u,v) \rightsquigarrow$ $u^2$, $\theta$, $q$

$x = u^2 \cos v$

$y = u^2 \cos v$

$ds^2 = dx^2 + dy^2 = dr^2 + r^2 d\theta^2$





$\theta$, $u$

$s_1, s_2 \perp$ coords

$\delta\hat{s}^2 = \delta\hat{s}_1^2 + \delta\hat{s}_2^2$

$\delta x^2 + \delta y^2$, $r$, $\delta y$, $\delta x$

$\delta\theta$, $\delta r$

New Length:

$\delta\hat{s}_1 = \delta r$

$\delta\hat{s}_2 = r\,\delta\theta$ arc length

$\Rightarrow$ $\delta\hat{s}_1^2 = \delta r^2$, $\delta\hat{s}_2^2 = r^2\delta\theta^2$

I. $u = r,\ v = \theta$ $du = dr$

$x = r\cos\theta$

$y = r\sin\theta$

$r\,d\theta = d$

orthogonal

$\Rightarrow \delta s^2 = \delta\hat{s}_1^2 + \delta\hat{s}_2^2$

$= \delta r^2 + r^2\delta\theta^2$

II. $r = u^2,\ \theta = v$ $\Rightarrow$ $dr = 2u\,du$

$x = r^2\cos\theta$

$y = r^2\cos\theta$

$d\theta = dv$

$\big\}$ algebra

$r\,d\theta = u^2 dv$

$\delta r = 2u\,du$, $r = u^2$

arc length $= r\,d\theta$

$= u^2 dv$

$d\theta$

coord length functions

$\hat{s}_1 = r,\ \hat{s}_2 = r\theta \Rightarrow$ $d\hat{s}^2 = d\hat{s}_1^2 + d\hat{s}_2^2 = dr^2 + r^2 d\theta^2 = \boxed{4u^2du^2 + u^4dv^2}$

$d\hat{s}_1 = dr,\ d\hat{s}_2 = r\,d\theta$

$$A = \frac{\partial \hat{s}_1}{\partial u} = \frac{\partial r}{\partial u} = 2u, \quad B = \frac{\partial \hat{s}_2}{\partial v} = \frac{\partial(r\theta)}{\partial v} = \frac{\partial(rv)}{\partial v} = r$$

$$\frac{\partial_v A}{B} = \frac{\partial_v(2u)}{B} = 0, \quad \partial_v\left(\frac{\partial_v A}{B}\right) = \partial_v(0) = 0$$

$u \perp v$

$$\frac{\partial_u B}{A} = \frac{\partial_u(r)}{2u} = \frac{\partial_u(u^2)}{2u} = \frac{2u}{2u} = 1, \quad \partial_u\left(\frac{\partial_u B}{A}\right) = \partial_u(1) = 0$$

$$K = \frac{-1}{2AB}\left(\partial_v\left(\frac{\partial_v A}{B}\right) + \partial_u\left(\frac{\partial_u B}{A}\right)\right) = \frac{-1}{2ur}(0 + 0) = 0$$