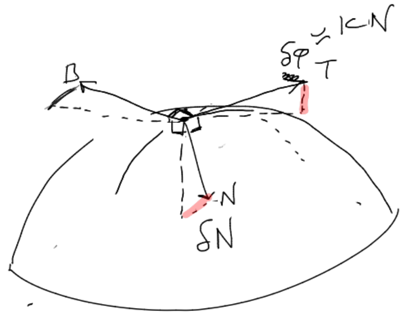
wk 5 - Wed



Last Time:

$$\boxed{\frac{dT}{dt} = \dot{T} = \kappa N}$$

$$\kappa = \frac{\partial \varphi}{ds}$$

B = binormal, perpendicular to both T & N

$= T \times N$

$$\boxed{\dot{B} = -\tau N}$$

(τ = torsion)
measures how much twisting

$$\boxed{\dot{N} = -\kappa T + \tau B}$$

Discovered ~ 1850, France

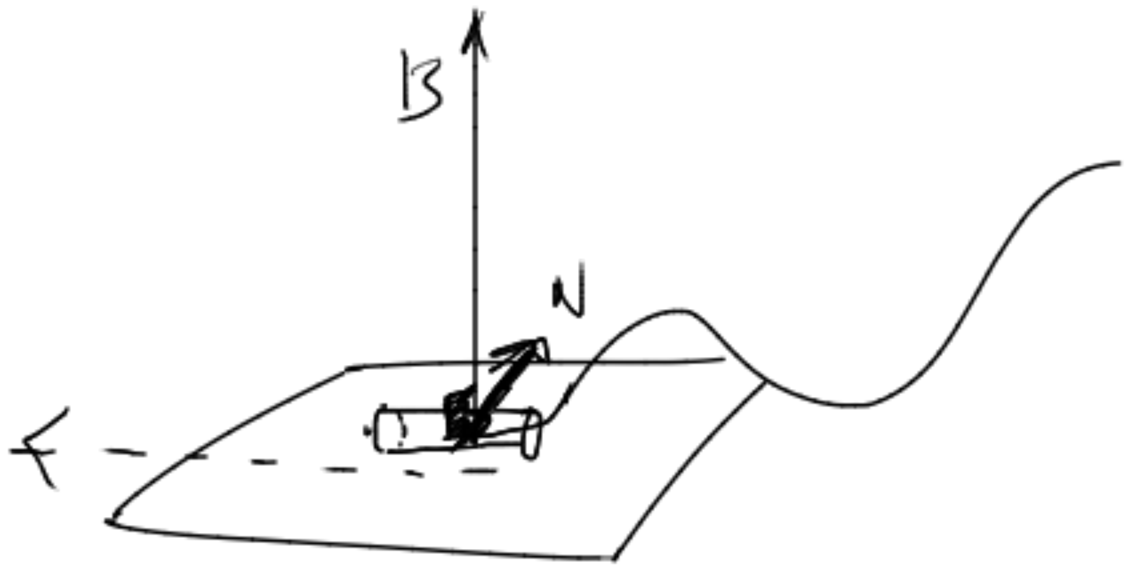
(N tips in direction perp. to both T & B)

$= \dot{N} \in \mathrm{Span}(T, B)$

Frenet-Serret Equations

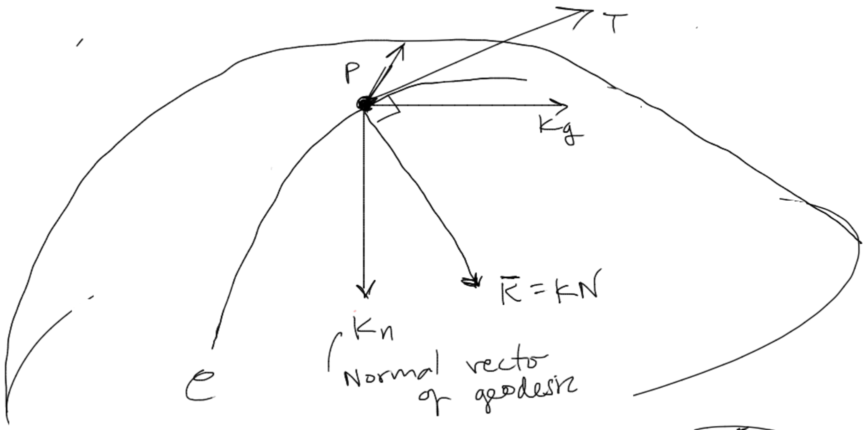
$$\begin{bmatrix} T \\ N \\ B \end{bmatrix}' = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \cdot \begin{bmatrix} T \\ N \\ B \end{bmatrix} = \Omega \begin{bmatrix} T \\ N \\ B \end{bmatrix}$$

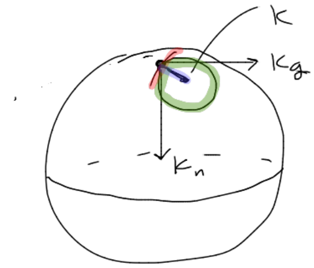
" Ω skew-symmetric . $\Omega^T = -\Omega$


B
N
F

Curvature Vector = Geodesic Curvature + Normal curvature

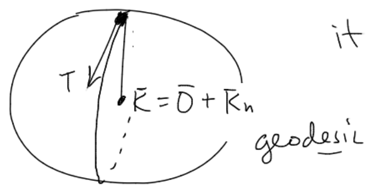
$\dot{T} = \kappa N = \bar{\kappa} = \bar{\kappa}_g + \bar{\kappa}_n$



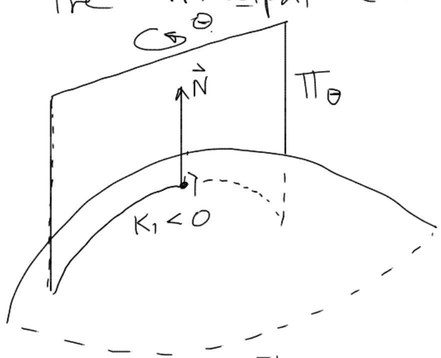


If the curve is a geodesic then it's "geodesic curvature" is $\bar{0}$.

$\bar{\kappa}_g$ = geodesic curvature captures the departure of the curve from being a geodesic



The Principal Curvatures of a Surface

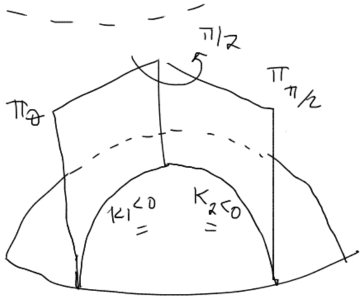


sign: does $\vec{N}$ point toward center of circle of curvature

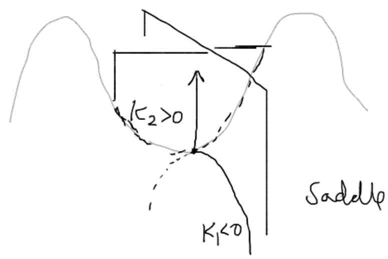
$K < 0 \Rightarrow$ No

$K > 0 \Rightarrow$ yes

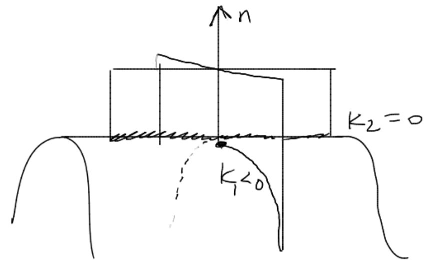
Maximum of curvature as you rotate
(always be $\pi/2$ apart



sphere

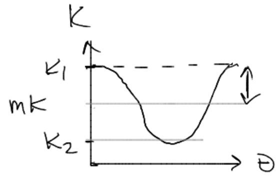


saddle



euclidean

K_i are functions



$\frac{K_1 - K_2}{2} = \frac{\Delta K}{2}$

mean curvature $= \frac{K_1 + K_2}{2}$

Euler: $K(\theta) = K_1 \cos^2\theta + K_2 \sin^2\theta$ using trig ID's

$$= \bar{K} + \frac{\Delta K}{2} \cos(2\theta)$$

While the notion of "geodesic curvature" is intrinsic ———
How much does a curve differ from a geodesic.

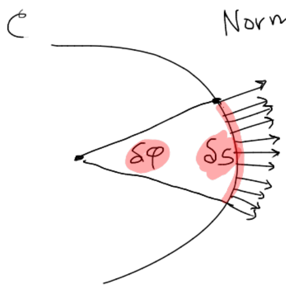
we also have notions "extrinsic curvature".

Extrinsic

$$\frac{dT}{ds} = \kappa N$$

$$\frac{dN}{ds} = -\kappa T$$

(Plane Curves)

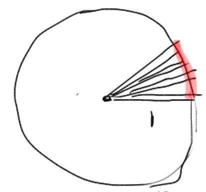


Normals on Surface

Map: $m : \mathcal{C} \to S^1$

translate / identify all vectors @ $\bar{0}$

$\xrightarrow{\delta\hat{s}}$

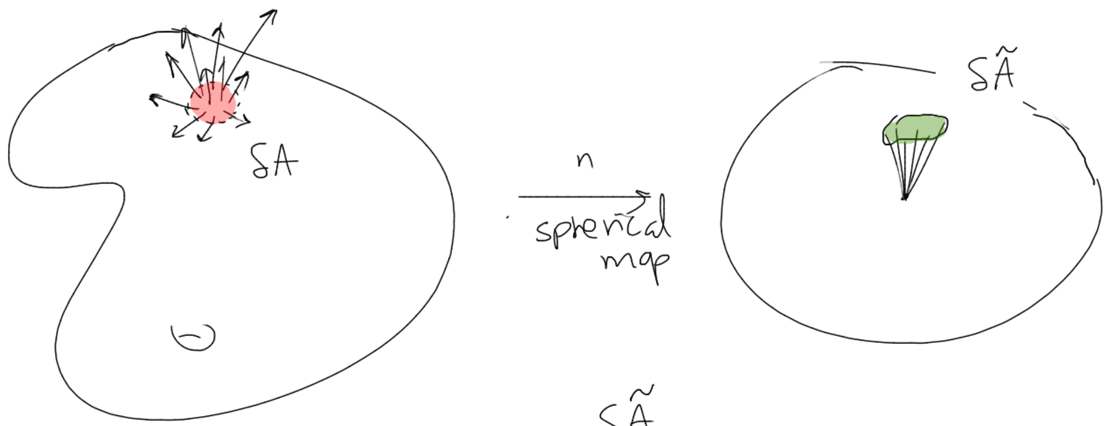


$\delta\hat{s} = \delta s$

since $\kappa = \dfrac{\delta\varphi}{\delta s} = \dfrac{\delta\hat{s}}{\delta s}$ can be viewed as local magnification factor.

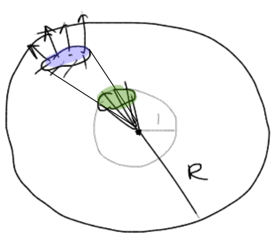
rate of spread

Extend to Surfaces



$$K_{ext} \approx \frac{\delta \tilde{A}}{\delta A}$$

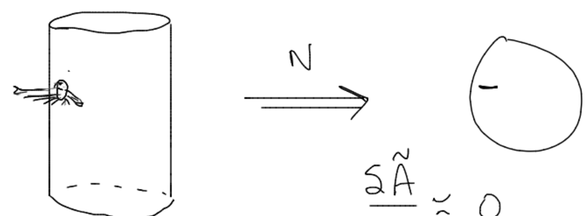
ex



Sphere radius = R

$$\frac{\delta \tilde{A}}{\delta A} = \frac{1}{R^2}$$

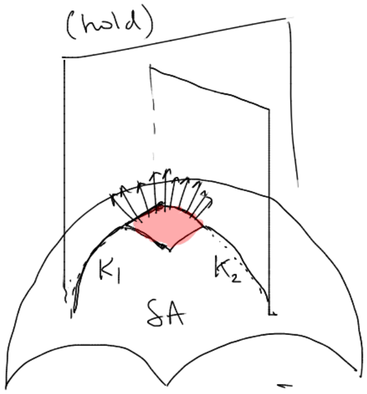
already know this as "curvature"



$$\frac{\delta \tilde{A}}{\delta A} \approx 0$$

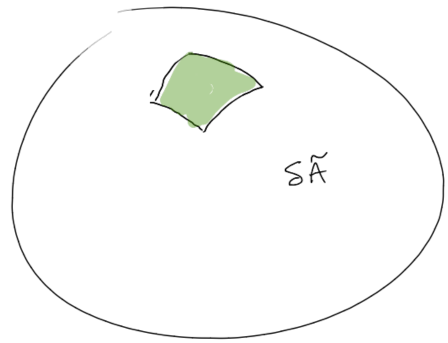
Prove Euler Formula

(hold)



scale by curvature

$\xrightarrow{n}$



$$\frac{\delta\varphi}{\delta s} = K$$

$$\delta\varphi = K\,\delta s$$

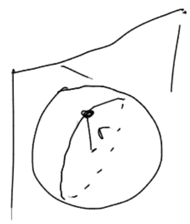
Rectangle ≈ length · width

$$\delta\tilde{A} = \varepsilon_1 \cdot k_1 \cdot \varepsilon_2 \cdot k_2$$

$r\,\delta\varphi$. =

$$K = \frac{\delta\tilde{A}}{\delta A} = \frac{\cancel{\varepsilon_1}\, k_1 \cdot \cancel{\varepsilon_2}\, k_2}{\cancel{\varepsilon_1} \cdot \cancel{\varepsilon_2}} = k_1 \cdot k_2$$

ex On sphere $k_i = \frac{1}{r}$

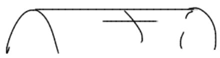


$k = \frac{1}{r}$

$$K = \frac{1}{r} \cdot \frac{1}{r} = \frac{1}{r^2}$$

Elliptic:  k_1, k_2 same sign : $K_{ext} = k_1 k_2$ positive

Hyperbolic  K_{ext} = negative since k_1, k_2 are of opposite sign

Parabolic :  $K_{ext} = k_1 \cdot k_2 = 0$ one $k_i = 0$

Umbilic [Euclidean]: $K_{ext} = 0 = k_1 \cdot k_2$, both are zero.