Wed. wk 7

Plan for Monday:

- Come prepared to talk about one of the old homework exercises. 5-10 min.
- Push written homework to Fri.

Last Time

$K(S) = \iint_S K\,dA$

Global Gauss Bonnett theorem:

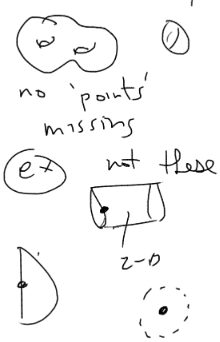
$K(S_g) = 4\pi(1-g) = 2\pi \cdot \chi(S_g)$

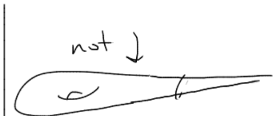
$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ 

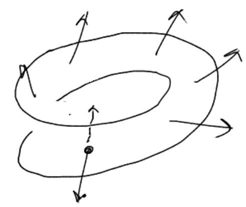
[0,1]

consistent choice of ordered basis (consistent choice of normal)

(closed, compact, orientable)







Genus g surface:

g = # of holes

what kinds of maps preserve "topology"?

homeomorphisms

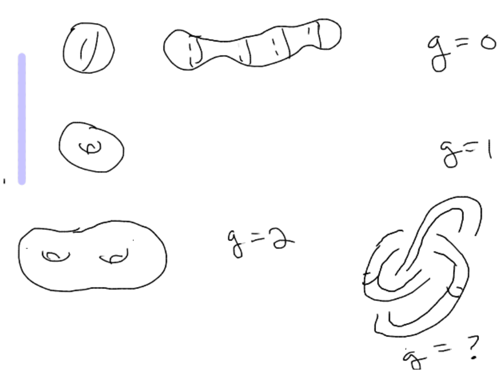
continuous function w/ continuous inverse

ex

what kinds of maps genus?

homotopy

continuous deformation thru time.



Connected Sum: surgery to combine surfaces

1. take two surfaces

2. remove disk from both

3. glue together along. or

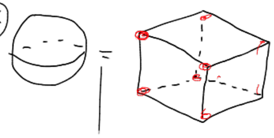
glue to tube

g=3

$\chi(S_g) = F - E + V$

top bottom

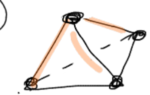
① Represent your surface as a simplicial complex. (Faces, Edges, Vertices)

ex 

homotopy

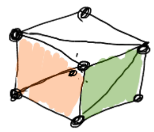
V = 8
F = 6
E = 12

$6 - 12 + 8 = 2$

ex 

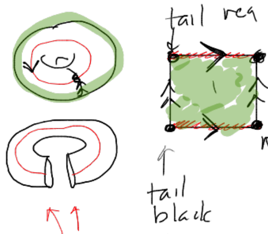
V : 4
F : 4
E : 6

$4 - 6 + 4 = 2$

② Triangulation : 

Faces
6 Squares
|
2 triangles

$\chi = 12 - 18 + 8$

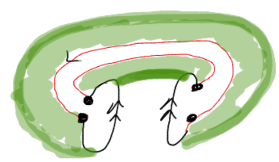
② 

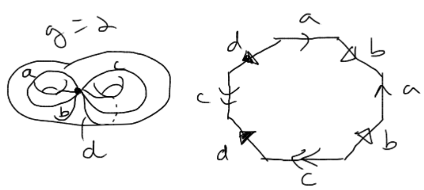
F : 1

E : 2 (1 red, 1 black)

V : 1

$F - E + V = 0$



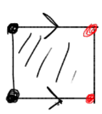
③ 

$\chi(\text{[drawing]}) = 1 - 4 + 1 = -2$

Formula

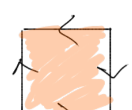
$\chi(S_g) = 2 - 2g = 2 - 2(2) = -2$

BTW

#1 

#2 

Need to upgrade simplicial complex

#3 

$\chi($  $) = 1 - 3 + 2 = 0$

$\chi(\text{circle})$ 

$\chi($  $) = 0 - 1 + 1 = 0$

$\chi($  $)$

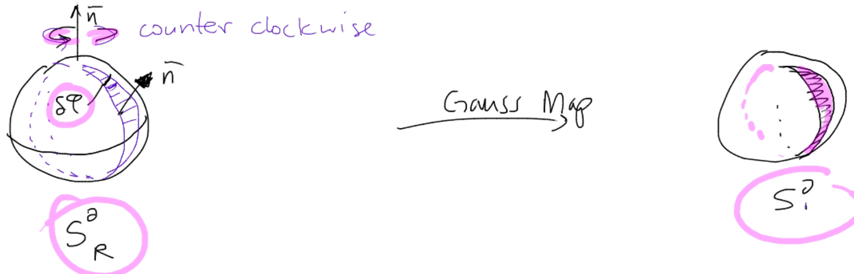
$= 1 - 3 + 2 = 0$

3 = labeled green orange

2 = red black

$\chi($ $) = 1$, $\chi(\cdot) = 1$, $\chi(\text{disk}) = 1$

Curvature



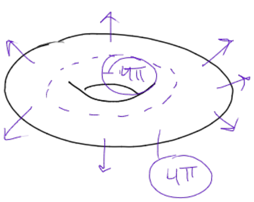
let great circle sweep around S^2_R.

as circle sweeps out angle $\delta\varphi$, the normal $\bar{n}$ covers area $\delta\varphi$ on S^2_1 $\Rightarrow$ In total, as circle sweeps across entire surface it's normal covers S^2_1 exactly once

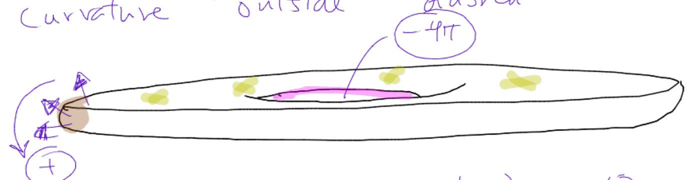
$$K = \frac{\delta\hat{S}}{\delta S} = \frac{4\pi}{4\pi R} = \frac{1}{R}$$

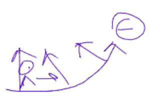
$\Rightarrow$ Total curvature : $K(S) = 4\pi = 2(2\pi)$

b/c as $\delta\varphi \to 2\pi$,



Total curvature "outside" dashed line = 4π





$K(T) = 0$