Requiz for those who will to take it in my office Tuesday from 9–12 or 1–2 or 3–4. will take 30 min max.

On your returned quiz, your final unit I score is in black.

MA-163 (W,08)
Quiz 1 on Integration
(Substitution & Parts)

Name ___Key___

As always, give all answers in mathematical sentences. Rewrite the integral and write your answers below the line and, if necessary, on the paper I've provided. Show your work as I've demonstrated in class; your work will be graded. Use correct notation -- like $\int \ldots\, d\ldots = \ldots$; $\ldots = \ldots\Big]_a^b = \ldots$; etc. Try the EC problem only after you've done the others.

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Do Problem 1 on this page below the line.

1. Do these by Parts: (a) $\overset{20}{}\int x\cdot\cos x\,dx$; and (b) either $\overset{20}{}\int \arctan x\,dx$ for full credit, or $\int \ln x\,dx$ for 3/4 credit.

1a) $\int x\cos x\,dx = x\sin x - \int \sin x\,dx$

$= x\sin x - (-\cos x) + C$

$= x\sin x + \cos x + C$

$\therefore \int x\cos x\,dx = x\sin x + \cos x + C$

Let $u = x$ & $dv = \cos x\,dx$
so $du = dx$ & $v = \sin x$

$D_x(\checkmark) = x\cos x + \cancel{\sin x} + \cancel{(-\sin x)}$
$= x\cos x$

b) $\int \arctan x\,dx = x\arctan x - \int \frac{x}{1+x^2}\,dx$

$= x\arctan x - \frac{1}{2}\ln(1+x^2) + C$

Let $u = \arctan x$ & $dv = dx$
so $du = \frac{1}{1+x^2}dx$ & $v = x$

$D_x(\checkmark) = x\cdot\cancel{\frac{1}{1+x^2}} + (\arctan x)\cdot 1 - \cancel{\frac{1}{2}\frac{1}{1+x^2}}$

$\int \ln x\,dx = x\ln x - \int x\cdot\frac{1}{x}\,dx$

$= x\ln x - \int dx$

$= x\ln x - x + C$

Let $u = \ln x$ & $dv = dx$
so $du = \frac{1}{x}dx$ & $v = x$

$D_x(\checkmark) = x\cdot\cancel{\frac{1}{x}} + (\ln x)\cdot 1 - \cancel{1}$
$= \ln x$

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Do these problems below the line.

20 2. Evaluate: $\int_0^{\pi/2} \cos^3 x \cdot \sin x \, dx$. (This is an eyeball substitution problem, not a Parts problem.)

20 3(a) Find, and simplify, this derivative: $D_x\left(x^n \sin x - n \cdot \int x^{n-1} \sin x \, dx\right)$;

20 (b) Then write the corresponding indefinite integral formula.

Note: If you can't do 3, you can do EC (a) below instead for less credit. (I counted them the same as 3 for credit.)

EXTRA CREDIT (Do only after you've done 1 - 3): Use Parts to find (a) $\int e^x \cdot \sin x \, dx$ or (b) $\int \cos^2 x \, dx$.

$$2 \quad \int_0^{\pi/2} \cos^3 x \sin x \, dx \overset{\text{scratch work}}{\underset{\text{below}}{=}} -\frac{1}{4} \cos^4 x \Big]_0^{\pi/2} = -\frac{1}{4}\left(\cos^4 \frac{\pi}{2} - \cos^4 0\right)$$ ★

$$= -\frac{1}{4}(0 - 1) = \frac{1}{4}.$$ ★

$$\boxed{\int \cos^3 x \sin x \, dx = -\frac{1}{4}(\cos x)^4 + C}$$

$$\therefore \int_0^{\pi/2} \cos^3 x \sin x \, dx = \frac{1}{4}$$

or (Formal Substitution)

$$\int_0^{\pi/2} \cos^3 x \sin x \, dx = \int_{w=1}^{w=0} w^3(-dw)$$

Let $w = \cos x$

so $\frac{dw}{dx} = -\sin x$

$\therefore \sin x \, dx = -dw$

When $x = 0$, $w = 1$

& when $x = \frac{\pi}{2}$, $w = 0$

$$= -\int_{w=1}^{w=0} w^3 \, dw = -\frac{1}{4} w^4 \Big]_1^0 = -\frac{1}{4}(0 - 1) = \frac{1}{4}$$ ★

3a) (20) $D_x\left(x^n \sin x - n\int x^{n-1} \sin x \, dx\right) = x^n \cdot \cos x + (\sin x) \cdot nx^{n-1} - n(x^{n-1} \sin x)$

$$= x^n \cos x + \cancel{nx^{n-1} \sin x - nx^{n-1} \sin x} = x^n \cos x.$$

(20) b) $\int x^n \cos x \, dx = x^n \sin x - n\int x^{n-1} \sin x \, dx$. See Comment (5). See Comment (6).

★ Linear presentation is sometimes okay, too, especially when space is at a premium.

Comments:

Comment ① Here is how you set out your algorithmic work:

Original Problem = 1st modification

= 2nd modification

= ⋮

= final modification.

The above would be sufficient, but it's even better if your last step is

∴ original problem = final modification.

Look at all the examples in Sections 7.1 + 7.2 — and elsewhere throughout the text. With very few exceptions, they all conform to the format above. From now on, I expect your work to conform to this format as well. (If you have to do scratch work, do it off to the side so it doesn't interrupt the flow of your exposition.)

In every discipline, there are conventions about using symbols, the meaning of terms, the style of exposition, etc. Learning a discipline involves learning the concepts and also learning how to express those concepts. Perhaps no one has ever taken the time to tell you how your work should be set out and why it should be set out that way. Instead, it's assumed you'll learn these things by emulating your texts. Apparently that's not sufficient. So from now on, you must make a conscious effort to write your work out as I've demonstrated on handouts and in class.

(continued)

① (Continued)

In some stylistic conventions, I (your teacher) have my own expectations that may be different from the text. In particular, I've shown you how I expect you to set out your work when you're doing Integration by Parts, Integration by Substitution, etc. I'll remind you again here using Probs 1a + 2.

② At the far right, you do the Parts nitty gritty

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx$$
$$=$$

Let $u=$ and $dv=$

so $du=$ + $v=$

① You write the original integral

④ Then you write the sequence of equal expressions (and they must be equal) as I showed you before.

And you put them in this arrangement:

③ Then you put the results back here.

For formal substitution, the format is the same. Look at my work on Prob 2 under (Formal Substitution)

Comment ②.

There will be no partial credit for relatively simple problems like these (i.e, where the final is easily differentiated) if there is no indication that you have tried to check your answer by differentiation.

Comment ③. It's not necessary to write $\ln|1+x^2|$. Instead, you can write $\ln(1+x^2)$ since $1+x^2>0$ for all x.

Comment (4)
When you do formal substition to evaluate a definite integral, you have to change the limit of integration. (See Assignment Sheet for Unit 2, Part 2 — The comments under Wed 2/6.)

Comment (5) Integration means finding an antiderivative. An antiderivative is a function you can differentiate to get the integrand. It doesn't make sense to learn to integrate if you don't know how to differentiate your result to check.

Comment (6) Basic definition:
$\int f(x)\,dx$ is the (family of) function whose derivative is f.

That is, (a) if $D_x(g(x)) = f(x)$, then $\int f(x)\,dx = g(x) + C$;
and (b) if $\int f(x)\,dx = g(x)$, then $D_x(g(x)) = f(x)$.

It is (a) that we use to do Problem 3:
Since $D_x\left(x^n \sin x - n\int x^{n-1}\sin x\,dx\right) = x^n \cos x$,
then $\int x^n \cos x\,dx = x^n \sin x - n\int x^{n-1}\sin x\,dx$.

Re the derivative, $D_x\left(n\int x^{n-1}\sin x\,dx\right) = n\,D_x\left(\int x^{n-1}\sin x\,dx\right)$
$= n\,x^{n-1}\sin x$.

This is (b) above: $D_x\left(\int f(x)\,dx\right) = f(x)$.

EC

a) $\int e^x \sin x\,dx = e^x \sin x - \int e^x \cos x\,dx$ Let $u = \sin x$ & $dv = e^x dx$ so $du = \cos x\,dx$ & $v = e^x$

$= e^x \sin x - \left[e^x \cos x - \int e^x(-\sin x)\,dx\right]$ Let $u = \cos x$ & $dv = e^x dx$ so $du = -\sin x\,dx$ & $v = e^x$

$= e^x \sin x - \left[e^x \cos x + \int e^x \sin x\,dx\right]$

$= e^x \sin x - e^x \cos x - \int e^x \sin x\,dx.$

$\therefore \int e^x \sin x\,dx = e^x(\sin x - \cos x) - \int e^x \sin x\,dx.$

Solve for $\int e^x \sin x\,dx$:

$2\int e^x \sin x\,dx = e^x(\sin x - \cos x) + C$ This is done in two steps.

$\therefore \int e^x \sin x\,dx = \frac{1}{2}e^x(\sin x - \cos x) + C.$

Check: $D_x\left(\frac{1}{2}e^x(\sin x - \cos x)\right) = \frac{1}{2}\left[e^x(\cos x - (-\sin x)) + (\sin x - \cos x)e^x\right]$

$= \frac{1}{2}\left[\cancel{e^x \cos x} + e^x \sin x + e^x \sin x - \cancel{e^x \cos x}\right]$

$= \frac{1}{2}(2e^x \sin x) = e^x \sin x$ ✓

½C

b) $\int \cos^2 x\,dx = (\cos x)(\sin x) - \int \sin x(-\sin x)\,dx$

Let $u = \cos x$ & $dv = \cos x\,dx$
so $du = -\sin x\,dx$ & $v = \sin x$

$= \sin x \cos x + \int \sin^2 x\,dx$

$= \sin x \cos x + \int (1 - \cos^2 x)\,dx$

$= \sin x \cos x + x - \int \cos^2 x\,dx.$

$\therefore \int \cos^2 x\,dx = \sin x \cos x + x - \int \cos^2 x\,dx$

so $2\int \cos^2 x\,dx = \sin x \cos x + x$

$\therefore \int \cos^2 x\,dx = \frac{1}{2}(\sin x \cos x + x) + C$