MA-163 (W,07)
Test 1 (Review, FTC, Applications of Def. Integral)

240 possible points. Name_______________________
DIRECTIONS are on another sheet of paper. Read the first before beginning.
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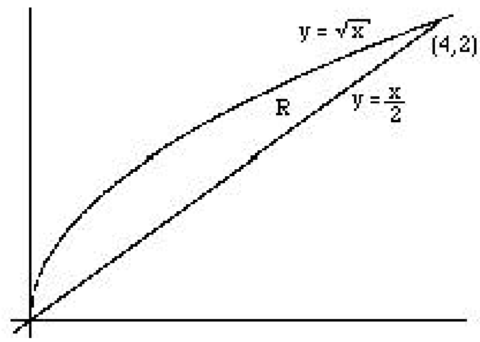
[20 each for Problems 1 - 3]
The first three problems relate to the region R shown at right -- the region in the first quadrant bounded on the top by the graph of $y = \sqrt{x}$ and on the bottom by the graph of y = x/2. You will probably have the redraw the region on your answer sheet for each problem. [Remember, no partial credit for an incorrect answer unless you have an appropriately labeled figure as I described in class. And please don't use a single picture for more than one problem.]

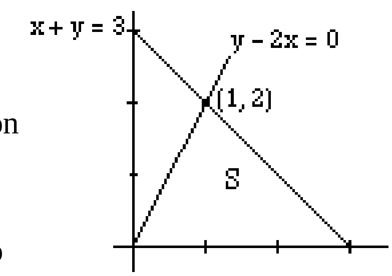


1. Find the area of the region R.

2. Find the volume of the solid obtained when R is revolved about the x-axis.

3. Write the definite integral that gives the volume of the solid obtained when the region R is revolved about the y-axis. You do not have to simplify the integrand nor integrate nor evaluate the definite integral.

[20 each for Problems 4 and 5]
The next two questions relate the the region S shown at right -- bounded by the x-axis and the lines x + y = 3 and y – 2x = 0.



4. Suppose that S is the base of a solid having the property that every cross section perpendicular to the y-axis is a square. Find the volume of the solid.

5. Write a definite integral expression that gives the volume of the solid obtained when the region S is revolved about the horizontal line y = –1. You do not have to simplify nor integrate nor evaluate the expression.

[40 possible]
The next problem relates to the region, T, described as follows: The top of T is bounded by $y = e^x$, the bottom of T is the line $y = \frac{x}{2}$, the left side is the y-axis, and the right side is the vertical line x = 2.

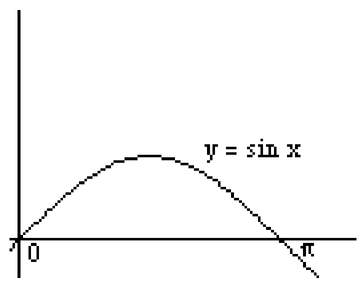
6. Find the volume of the solid obtained when T is revolved about the x-axis.

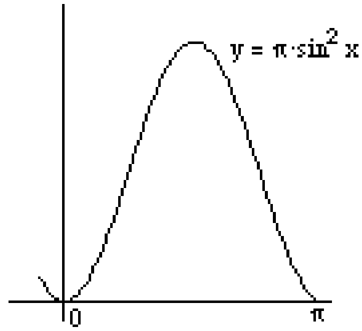
[40 possible: 30 for a; 5 each for b and c]
7. A particle moves along the curve $y = 2\cdot x^{3/2}$. At time 0, the particle is at the origin, and four seconds later it is at the point (4 , 16). (a) Find the distance the particle traveled along the curve during its three second trip. (b) What is the average speed of the particle over that time interval? (c) How far (as the crow flies) from its starting point is the particle after four seconds?

[30 possible: 10 each]
8a. Write a definite integral expression for the volume of the solid obtained when the region bounded by one arch of the graph of $y = \sin x$ and the x-axis is revolved about the x-axis.

b. Explain carefully how the question in (a) is related to the following question:

What is the area of the region above the interval $[0, \pi]$ and below the graph of $y = \pi \cdot \sin^2 x$?





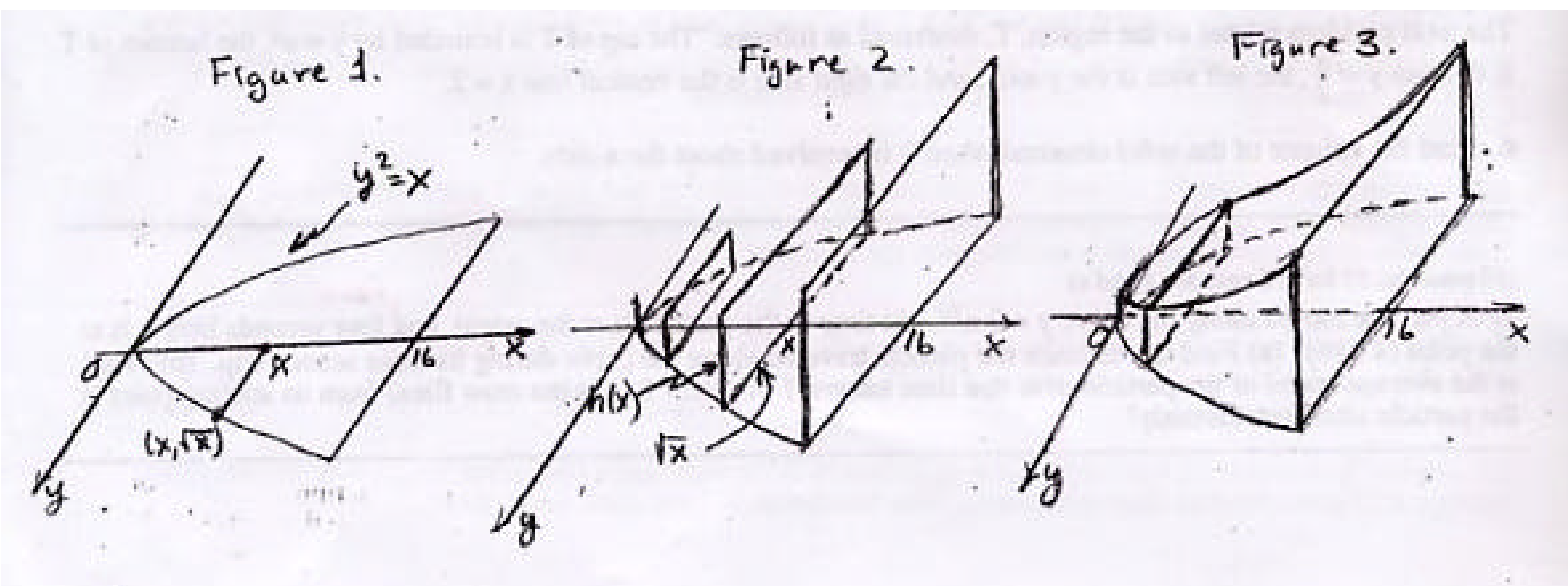
c. Even though it's unlikely that, at this point, you can find an antiderivative to compute the definite integral you wrote in (a), you can still give an approximation for the volume of the solid in (a). Approximate the volume to four decimal places. Tell what you did to get your value.

[30 possible]
9. The base of the solid shown in Figure 3 below is a parabolic region bounded by $y^2 = x$ and the line $x = 16$. (Figure 1.) Cross sections of the solid perpendicular to the x-axis at the coordinate $(x, 0)$ are rectangles whose heights, $h(x)$, depend on x. (See Figure 2.) There is no explicit expression for the function, h. Instead, measurements have been taken every four feet along the x-axis with values given in the table below.

x	0	4	8	12	16
h(x)	2	3	3	4	6

a. Write a definite integral that expresses the volume of the solid. Your integral will involve $h(x)$ along with other things.

b. Using trapezoids, approximate the volume of the solid, showing your work.



EXTRA CREDIT problems. EC helps your test score, but nowhere near as much as the required problems. Therefore, you should do the EC problems only after you've completed, and checked over, everything else you can do on the test. Each question is worth 30 EXTRA CREDIT points on this test.

The following question is about the Fundamental Theorem of Calculus (FTC).

10. Assume that f is a function that is continuous on [a, b], and that all the other symbols and expressions below have their standard meaning. Each of the statements on the left is either false or can be justified using one of the reasons from the column on the right. Some of the reasons from the right hand column may be used more than once, and some may not be used at all. **Put the number of the appropriate reason in the blanks.**

____ $\int_a^b f(x)dx = F(b) - F(a)$ where F is an antiderivative of f.	1. By part of the FTC.
____ The derivative of an antiderivative of the function f is f.	2. By definition
____ An antiderivative of the derivative of the function f is f.	3. This is false.
____ $\int_a^b f(x)dx = \lim_{\text{Max } \Delta x \to 0} \left[\sum_{k=1}^{n} f(x_k^*)\Delta x_k\right]$.	
____ $\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$.	
____ $\lim_{\text{Max } \Delta x \to 0} \left[\sum_{k=1}^{n} f(x_k^*)\Delta x_k\right] = \int f(x)dx \Big|_{x=a}^{x=b}$	

11.(a) Show that $\frac{d}{dx}\left(\frac{1}{2}(x - (\sin x)(\cos x))\right) = \sin^2 x$. Obviously, you have to show your work in detail. You will probably have to use one of the trig identities on the Precalculus Formula Sheet.

(b) Use the formula in (a) to find the exact volume of the solid described in Problem 8a. Compare this with your answer in 8c.

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12. Referring to the region T described in Problem 6 -- the region in the first quadrant bounded by $y = e^x$, the line $y = \frac{x}{2}$, the y-axis, and the vertical line x = 2. Suppose T is the base of a solid having the property that every cross section perpendicular to the x-axis is a semicircle. Write the definite integral expression that gives the volume of this solid. you do not have to simplify nor integrate nor evaluate.

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13. Referring to Problem12, suppose T is the base of a solid having the property that every cross section perpendicular to the y-axis is a square. Write the definite integral expression that gives the volume of this solid. Again, you do not have to simplify nor integrate nor evaluate.

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14. Derive the formula for the volume of a sphere with radius r by revolving the graph of $y = \sqrt{r^2 - x^2}$ about the x-axis. You have to show your work in detail.

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15. Derive the formula for the volume of a right circular cone with base radius r and height h by revolving an appropriate figure about an appropriate axis. You have to show your work in detail.