MA-163 (W,07)
Test 2 (Integration Techniques)

The instructions are on a separate sheet. Read them first. Name _____

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[20 possible]

1. Here are two easy substitution problems to warm up on: (a) $\int \left(e^{1-\cos x}\right)\sin x\,dx$; (b) $\int_0^1 \frac{x^3}{\sqrt{x^4+1}}\,dx$.

[20 possible]

2. Write the given fraction as the sum of partial fractions as if you were going to integrate it by the method of Partial Fractions. You must determine the coefficients. (The coefficients are whole numbers and should be easy to find. If they're not, you're doing something wrong.) After you've determined the constants, be sure to write the fraction as an appropriate sum of partial fractions. Do not integrate. Here is the fraction: $\frac{x^4+x^3+3x^2+2x+1}{x(x^2+1)^2}$.

Alternatives: $\frac{2x^2-3x+2}{x\cdot(x^2+1)}$ for less 75% credit; $\frac{4x+13}{x^2-x-6}$ for 50% credit.

[10 possible]

3. Noting the degrees of the numerator and denominator, find $\int \frac{x^3+2x+1}{x^2+1}\,dx$.

[20 possible]

4 a. Integrate by Parts: $\int \cos^2 x\,dx$ to get $\frac{1}{2}[x + (\sin x)(\cos x)] + C$. Obviously you must show your work.

b. Using the identity $\sin^2 ß + \cos^2 ß = 1$ and formula for $\int \cos^2 x\,dx$ above., derive, and simplify, the formula for $\int \sin^2 x\,dx$. [Obviously, you must show your work in detail .]

[10 possible]

5. Remembering that $\tan^2 ß + 1 = \sec^2 ß$, find $\int \tan^2 x\,dx$.

[You'll be able to use this result in a later problem so raise ask me if you're not able to get it or if you're uncertain about your result. For a few points off, I'll check your answer now. If it's wrong, I'll give you the correct answer. A wrong answer means all points off, of course.]

[10 possible]

6. Do this by Trigonometric Substitution: $\int \frac{\sqrt{x^2-1}}{x}dx$.

[20 possible]

7. Do these by Parts: (a) $\int x\cdot\cos x\,dx$; (b) $\int \arctan x\,dx$.

[20 possible]

8. Do this by Trig Substitution: $\int \frac{dx}{(x^2+a^2)^2}$.

The last two required problems are on the back of the paper. Don't forget to do them!

[20 possible]

9. Find these any way you choose: (a) $\int \frac{4x-3}{x^2+x}dx$; and (b) $\int \frac{dx}{x^2+2x+2}$.

[15 possible]

10. Refer to the Short Table of Integrals provided with this test to find this integral: $\int 3\sqrt{3+4x^2}\,dx$. Simplify your result as much as possible.

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The following problems are for EXTRA CREDIT. Do them only after you've completed, and checked, you can do on problems 1 - 10.

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[20 possible]

11. Find this integral by Partial Fractions, showing your work in detail: $\int \frac{dx}{x^2-a^2}$.

[10 possible]

12. Check your result in Problem 11, showing your check explicitly.

[20 possible]

13. Find the integral in Problem 11 using Trigonometric Substitution.

[30 possible]

14 (a) Derive a reduction formula for $\int u^n \cos u\,du$. (b) Check your formula, showing your check explicitly.

(c) Then use your formula to show you got the right result for Problem 7a.

[30 possible]

15. The hyperbolic functions sinh(x), cosh(x),and tanh(x) are defined as follows:

$\sinh(x) = \frac{e^x - e^{-x}}{2}$ $\cosh(x) = \frac{e^x + e^{-x}}{2}$ $\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$

a. Find the derivatives of these three functions, giving your results in terms of hyperbolic functions. [You have to derive the formulas; you can't just use the analogous formulas for sine, cosine, etc.]

b. Use your results from (a) to find the integrals of the first two of the hyperbolic functions.

[30, 20, or 10 possible]

16. From Problem 2, find the integral of the rational fucnction that you wrote as the sum of partial fractions.