MA-163 (W,07)
Test 3 (Potpourri 1)

[140 possible points: 20 points for each question.] Name ________________________

The instructions are on another sheet. **Read them first**.

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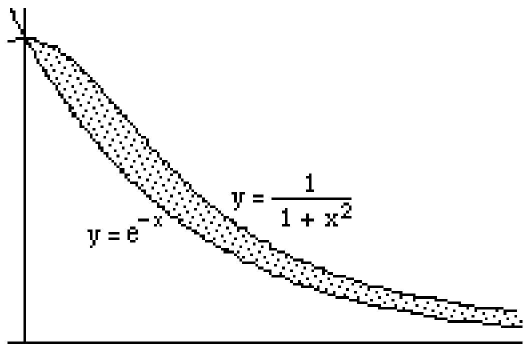
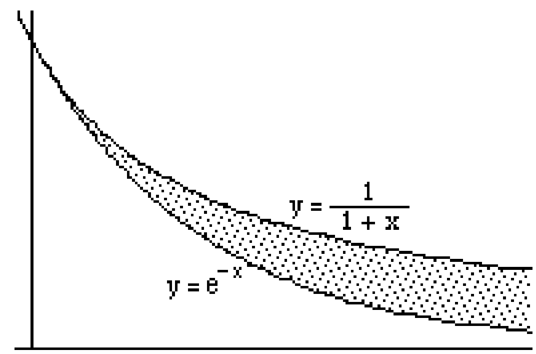
1a. Find the area of the infinitely long region between the graphs of $y = \frac{1}{1+x^2}$ and $y = e^{-x}$ shown below left.

b. Find the area of the infinitely long region trapped the graphs of $y = \frac{1}{1+x}$ and $y = e^{-x}$ shown below right.

[Correct notation and logic are important in doing these problems.]





Note: If you can't do the problems above, you may find these improper integrals for a maximum of 80% credit:

a. $\int_0^\infty e^{-2x}dx$ b. $\int_1^\infty \frac{x}{x^2+1}dx$ (Note the lower limits of integration.)

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2. Use L'Hospital's Rule to find this limit: $\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^x$ where k is a positive constant. Note: If you can't do this limit, you may, for a maximum of 85% credit, use L'Hospital's Rule to find $\lim_{x\to 0}\frac{1-\cos x}{5x^2}$.

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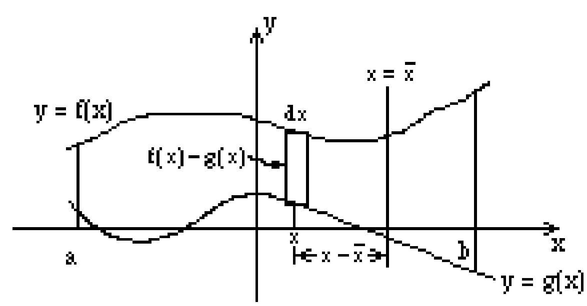
3. Suppose a lamina of uniform thickness with density ρ is the plane region bounded by y = f(x), y = g(x), x = a, and x = b where f(x) ≥ g(x) for all x in [a, b]. The moment about the line $x = \bar{x}$ is defined to be

$M_{x=\bar{x}} = \int_a^b (x-\bar{x})\rho\big(f(x)-g(x)\big)dx$. The x-value of the centroid of the lamina is defined to be the value of $\bar{x}$ that makes $M_{x=\bar{x}}$ equal to zero. The formula for $\bar{x}$ is

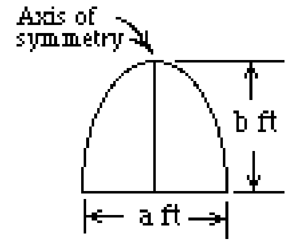
$$\bar{x} = \frac{\int_a^b x\cdot\big(f(x)-g(x)\big)\,dx}{\int_a^b \big(f(x)-g(x)\big)\,dx}$$



Show how this formula is obtained from the definitions of moment and centroid given above. Show all the steps in your derivation.

Notes: You may use this formula in the subsequent parts of the test. The formula for $\bar{y}$ is given on the formula sheet.

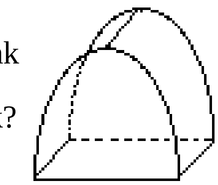
The rest of the questions 4 - 6 below involve the figure at right. The figure has a vertical axis of symmetry and has the dimensions shown.



Note: All of your solutions must involve a coordinate system and an equation for the figure. If you cannot get a coordinate system and an equation, ask me and I'll give them to you for 8 points off. Also, if you can't get any of the problems 4 - 6, you may do the corresponding alternative problems at the bottom of the page.

Here are questions about this figure:

4. The figure above is a <u>semi-ellipse</u> with a = 2 and b = 2. It is the front of a 3 foot long tank shaped like half of an oil drum lying on its flat bottom. The tank is filled with a fluid that weighs 60 pounds per cubic foot. What is the total hydrostatic force on the front of the tank?



Note: If you can't answer (4), you may, for a maximum of 80% credit, do Alternative Problem 4 below.

5. The figure above is a metal plate, again shaped like a <u>semi-ellipse</u> with a = 2 and b = 2. The centroid of the plate is on the vertical axis of symmetry. How far up from the bottom of the plate is the centroid? You may assume that the figure has uniform thickness and uniform density.

Note: If you can't answer (5), you may, for a maximum of 80% credit, do Alternative Problem 5 below.

6. The figure described above is a piece of a <u>parabola</u> with a = 2 and b = 2. A solid is obtained by revolving the <u>right half of the figure</u> about the vertical axis of symmetry. What is the surface area of the resulting solid?

Note: If you can't answer (6), you may, for a maximum of 80% credit, do Alternative Problem 6 below.

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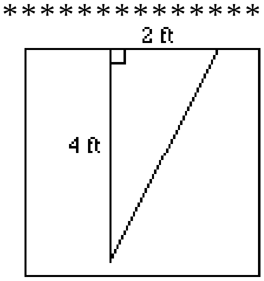
Alternative Problems:



Alt. Prob 4. The triangular plate at right is submerged in water. Set up an integral that expresses the total hyrostatic force on the plate. Then find this force. (Water weighs 62.5 pounds per cubic foot.)

Alt. Prob. 5. Find the centroid of the region bounded by $y = \sqrt{x}$ on the top, the x-axis on the bottom, and the vertical line x = 9 on the right. Draw an accurate graph of the figure and show the centroid.

Alt. Prob. 6. Find the surface area of the solid obtained by revolving the curve $y = \sqrt{x}$ for $0 \le x \le 9$ about the x-axis.

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EXTRA CREDIT: Do these only after you have completed, and checked, everything else you can do on the test.

7. The formula for the y-value of the cetnroid of a region is given on the formula sheet. It is obtained by first defining the moment, $M_{y=\bar{y}}$, about the line $y = \bar{y}$, and then solving the equation $M_{y=\bar{y}} = 0$ for $\bar{y}$. Your problem is to draw a picture (You may use my figure in Problem 3 but include the line $y = \bar{y}$ and whatever else is appropriate.) and then write the formula for $M_{y=\bar{y}}$. After that, solve the equation $M_{y=\bar{y}} = 0$ for $\bar{y}$, showing your work.

8. For the tank in Problem 4, how much work is done by pumping the fluid out of the tank using a pump that is 3 feet above its top?