

MA-163 (W,07)
Test 4 (Potpourri 2)

Name _____

[150 possible points]

Do the problems on the paper provided, numbering the problems clearly. Answer questions in sentences. Be sure your name is on each paper. **Neatness, notation and logic all count.**

[20 possible points]

1. This problem is about the normal probability distribution. You may **not** use MAPLE nor any other computer algebra system to answer this question.

Suppose you know the following things about the standard normal probability density function:

$$A. \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx = 1 \quad . \quad \int_{-\infty}^{\infty} x \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx = 0 \quad C. \int_{-\infty}^{\infty} x^2 \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx = 1$$

Use these facts, together with the properties of integrals, to find the value of the following integral:

$$\int_{-\infty}^{\infty} (x - r)^2 \cdot \frac{1}{s\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-r}{s}\right)^2} dx. \text{ You must show all your work in detail.}$$

If you can't find do the previous integral, you may do this one for less credit: $\int_{-\infty}^{\infty} x \cdot \frac{1}{s\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-r}{s}\right)^2} dx$

[30 possible points, 10 each]

2. These questions relate to the random variable X that can take on values from the interval $[0, 2]$ and has this density function: $f(x) = \frac{3}{4}x^2(2 - x)$ for x in $[0, 2]$. [The relevant definitions are on the formula sheet.]

- Verify that f is a probability density function by showing that it satisfies conditions (1) and (2) in the definition.
 - Find the probability that X takes on a value between 1 and 2; i.e., find $P(1 < X < 2)$.
 - Find the mean, μ , of the distribution. [The definition is on the formula sheet.]
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[50 possible points: 15, 15, 15, 5]

3. A particle is moving along a curve in the plane. Its position t seconds from a base time is given by the parametric equations $x = t - 2$, $y = 6 + 2t - \frac{1}{2}t^2$.

- Give the Cartesian equation of the curve along which the particle is moving. Draw a figure showing the path of the particle. Show the position of the particle for integral values of t with $0 \leq t \leq 6$. Show the direction it's moving with arrows. [You'll have to draw your own coordinate system using centimeters as your units. Your graph must be accurate!]
 - Give the slope of the line tangent to the path at the time when $t = 4$. Draw this line on your graph for (a).
 - What is the length of the arc between the points where $t = 2$ and $t = 6$? [Remember, the units are in centimeters.]
 - What is the average speed of the particle between times $t = 2$ and $t = 6$? [Remember, t is measured in seconds.]
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[50 possible points: 10, 25, 5, 5, 5]

4. Obviously, when a cooler object is put in a warmer environment, the object warms up. It is an empirical fact that the rate at which the object warms is directly proportional to the difference between the temperature of its surrounding medium and its current temperature. Suppose an object is immersed in boiling water which has a constant temperature of 100°C (so in this situation, the rate is directly proportional to 100 minus the object's current temperature.) The object has an initial temperature of 40°C; ten minutes later, its temperature is 85°C. You are to derive an equation that gives the temperature of the object as a function of the time it has been in the water.

- a. Define variables and state the conditions given above in terms of a differential equation with initial conditions.
- b. Solve the differential equation to obtain an equation for the temperature of the body as a function of time. You must show your work in detail. [Note: if you can't get an equation, do (a) and ask me for an equation so you can do parts c thru e.
- c. Use your equation to determine when the temperature of the body was 75°C. Give your answer to the nearest second. Show your work.
- d. Draw a graph of the function for values of time from 0 to 30 minutes.
- e. When will the temperature of the body reach 100°C?

The following problems are for EXTRA CREDIT. Do them only after you've done all you can on Problems 1 - 4 and have checked these problems over.

[30 possible points]

5. A radioactive substance decays at a rate that is proportional to the amount present at any time. At exactly 11 a.m., a careful weighing gave 5.0 mgs of the substance; a second weighing at exactly 12:30 p.m. gave 1.0 mgs.

- a. Define variables and state the conditions given above in terms of a differential equation with initial conditions.
- b. Solve the differential equation to obtain an equation for the amount of substance remaining as a function of time. You must show your work in detail.
- c. Finally, give the half-life of the substance correct to the nearest minute.

[30 possible points]

6. A curve is the graph of the parametric equations $x = e^t \cdot \sin t$, $y = 10 + e^t \cdot \cos t$ for $0 \leq t \leq \pi$.

- a. Find the value of t, and the point or points on the curve, where the tangent line to the curve is vertical.
- b. Find the value of t, and the point or points on the curve, where the tangent line has slope 0, i.e., is horizontal.
- c. Draw a graph of the curve and show that your solutions are correct.

[30 possible points]

7. A particle is moving along a line. Its position t minutes after a base time t_0 is given by the parametric equations $x = -2 + 2t$, $y = 12 - 2t$. A second particle is moving along the curve whose parametric equations are given in Problem 3. Its position is given by those parametric equations where time is measured from the same base time, t_0 . When will the particles be closest to one-another? (This is an extreme value question.) Where will they be at that time? How far apart will they be then?