

MA-161 (F,06)
Unit 1 Test

[150 possible points]

Problem 1a is on a separate sheet. You'll do it first.

Name _____

[20 possible]

1. Problem 1a is on the separate sheet that you already turned in. Problems 1b and 1c are on the last page of the test. Do those problems now. Give your work and answers on that page.

[10 possible]

2. Simplify: $\csc^2(\tan^{-1}x)$. [If you can't do this one, you may simplify $\sin(\arccos \frac{a}{b})$ for less credit.]

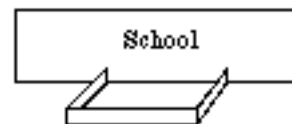
[20 possible]

3. If $f(x) = \sqrt{x}$, use the limit rules to derive $\lim_c \frac{f(x) - f(c)}{x - c}$, showing all your steps and justifying each step.

If you can't do the limit above, you may, for 90% credit, use the limit rules to find $\lim_x \frac{\sqrt{x} - 3}{x - 9}$. Again, give reasons for each step. If you can't do either of the first two, you may find, for 80% credit, $\lim_a \frac{x^2 - a^2}{x - a}$, again showing all your steps and giving reasons. If you can't do any of the first three, you may, for 70% credit, use the limit rules to find $\lim_x \frac{x^2 - 9}{x - 3}$. (If you're unsure about these, you may do more than one and I'll count the answer that gives the most points.)

[20 possible]

4. The school board has provided you with 220 linear feet of fencing material. You must fence in a rectangular playground that uses the school as one side of the playground. [See the figure at right.] The goal is to determine the dimensions of the playground so that the area is as large as possible.



i) Write an equation that expresses the area of the playground as a function of the length of the sides perpendicular to the school. [Please note: You will have to define the variables you use in this problem.]

ii) Use your grapher to draw a graph of the function. [Note: If you didn't get a formula for the area, ask me and I'll give you the equation for points off so you can finish the problem. Define the variables first before you ask me for the equation.] Reproduce the graph on your answer sheet. Use an appropriate viewing rectangle and label the axes. Then use the graph to answer these questions:

iii) Finally, answer these questions: How long should the perpendicular sides be so that the area of the playground is as large as possible? In this case, how long is the remaining side and what is the area of the playground?

[20 possible]

5. Within about 10 kilometers of the earth's surface, atmospheric pressure is approximately a linear function of altitude. (Pressure depends on other factors, too, which we'll ignore here.) Suppose that at an altitude of 2 km the pressure is 12.5 lbs per square inch, and at 5 km, the pressure is 9.2 lbs/in².

a. Define variables and write an equation that gives the atmospheric pressure as linear function of altitude.

b. What is the numerical value of the slope of the line? Explain what this number means as a rate of change in the context of this problem, using the appropriate units of measurement for the slope.

[20 possible]

6. The functions f, g and h are defined as follows: $f(x) = 6 - x$ $g(x) = x^2$ $h(x) = \begin{cases} x + 5 & \text{if } x < -1 \\ x^2 & \text{if } -1 < x < 2 \\ 6 - x & \text{if } x > 2 \end{cases}$

For these functions, find and simplify the following:

- (a) $f(g(x))$ (b) An equation for $g \circ f$. (c) $\frac{f(a+w) - f(a)}{w}$ (d) $\frac{g(t) - g(x)}{t - x}$ (e) $h(0)$ (f) $h(2)$ (g) $h(-1)$.

[20 possible]

7. For the function h given in Problem 6 above, give the value of each of the following limits, if it exists. If it doesn't exist, say so. (A graph might help. If you can't draw a graph, ask me and I'll draw one for you for points off. Also, be sure to answer in (mathematical) sentences -- like $\lim_{x \rightarrow a} f(x) = b$.)

- (a) $\lim_{x \rightarrow 2^+} h(x)$ (b) $\lim_{x \rightarrow 2^-} h(x)$ (c) $\lim_{x \rightarrow 2} h(x)$ (d) $\lim_{x \rightarrow -1^+} h(x)$ (e) $\lim_{x \rightarrow -1^-} h(x)$.

[10 possible]

8. Use graphical methods to approximate the value of $\lim_{x \rightarrow 1} \frac{\arctan x - (\pi/4)}{x - 1}$. Give your result correct to three decimal places. Show how you got the result by drawing the appropriate graph. (Be sure to include enough parentheses when you put the function in your calculator.)

[10 possible]

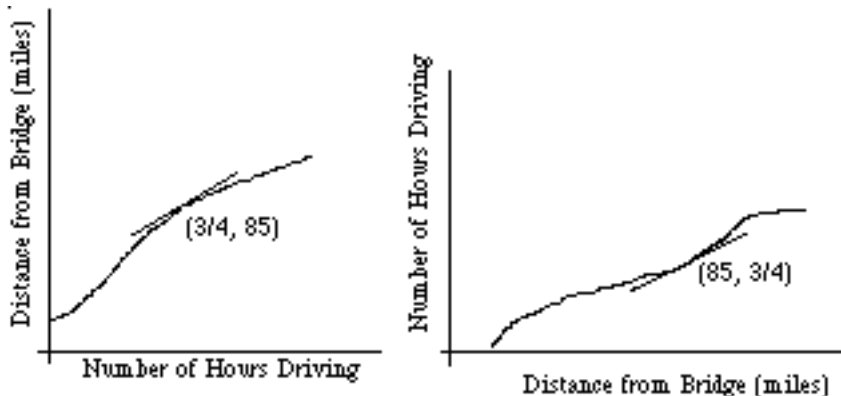
9. Use a table of values to approximate this limit: $\lim_{x \rightarrow 0} \frac{3^x - 1}{x}$. You must have a total of at least eight entries in your table. (Be sure to include enough parentheses when you put the function in your calculator.)

These problems are for **EXTRA CREDIT**. Do them only after you've completed, and checked over, everything else you can do on the test. (Please note that the Extra Credit is far less helpful to your test score than just finding even a small error in one of the required problems! **So check over Problems 1 - 9 before doing the EC problems.**)

10. If $g(x) = x^2 + 1$, find $\lim_{h \rightarrow 0} \frac{g(a+h) - g(a)}{h}$.

11. Suppose I leave Indian River, a town 20 miles south of the Mackinaw Bridge, and drive south on I-75. The first graph at right might be the graph of the distance I am from the bridge as a function of the number of hours I've been driving.

The inverse of this function interchanges all the ordered pairs on the graph. The graph of the inverse function might be like the picture at the far right. Suppose I estimate the slope of the tangent to the left graph at the point $(3/4, 85)$ to be 56. I also estimate the slope of the tangent to the right graph to be .02.



Explain in detail what each of the two slopes means in the context of this situation, using the appropriate units of measurement for the slopes in each case.

12. On the coordinate system below, draw an accurate graph of the function defined by the equation $y = 2^x$. Then draw an accurate graph of the inverse of the function defined by this equation. (You want the **inverse function**, not the reciprocal.) [Do all of this problem on this page below the graph.]

(a) What is an equation for the inverse function? Give your answer in the form $y = \underline{\hspace{2cm}}$.

(b) At the point $(0,1)$ on the graph of $y = 2^x$, draw carefully the tangent line and estimate its slope. Show how you're estimating the slope.

(c) At the point $(1,0)$ on the graph of the inverse function, draw the tangent line and estimate its slope. Again, show your work.

