MA-161 (F,06)
Test 4 (Integration and FTC)

210 possible points; 30 for each problem 1 -7. Name ____________________________

The general directions are on page 3. Read them first.

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[10 each]

1. Give the general antiderivatives of the given expressions. For each, x is the independent variable. Write the problem and your answer using correct "indefinite integral" notation. [Like $\int \sec^2 x\, dx = \tan x + C$.]

a. $ax^2 + bx + c$ b. $\frac{e^x}{2} + 2\cdot\sin x$ c. $\frac{1}{x} + \sqrt{x} + \frac{1}{\sqrt{x}} + \frac{1}{\sqrt{1-x^2}}$

[10 each]

2. Evaluate these definite integrals. Give the exact result. Write the problem and show your work, using notation correctly.

a. $\int_0^1 (e^x + 2x)\, dx$ b. $\int_1^3 \frac{2}{x}\, dx$ c. $\int_1^4 \sqrt{u}\left(\frac{1}{u} + 1\right) du$

[20 pts for the first question; 10 pts for the second]

3. Compute $\int_0^w kx\, dx$ (where k is a positive constant) in two different ways, first by using the **definition of definite integral**, and, second, by using the **Fundamental Theorem of Calculus (Part 2).** Tell which is which.

[10 for a; 5 for b; 15 for c]

4. (If you understand the FTC, and you know an antiderivative of 1/x, this question should take no more than a few minutes.)

Draw the graph of $f(x) = 1/x$ for $x > 0$ and consider the interval [1, 3].

Suppose we partition [1, 3] into n equal subintervals, each of length $(3 - 1)/n = 2/n$. We then use right-hand endpoints to form the n^{th} Riemann sum: $R_n = \sum_{i=1}^{n} \frac{2}{n}\cdot f\left(1 + i\cdot\frac{2}{n}\right)$.

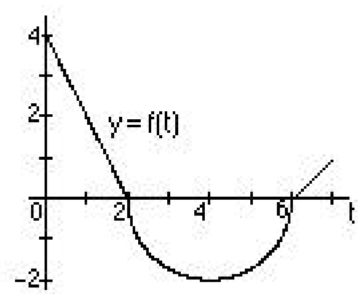
a. On your graph, draw a picture of the situation when n = 4. Then tell exactly the value of R_4.

b. You have the means to approximate R_{25}. Find this value rounded to the third decimal place. Tell how you got your value.

c. **[This is the important part of this problem.]** Finally, we consider $\lim_{n\to\infty} R_n$. What is the **exact** value of this limit? Tell how you can find the value **without actually having to go through the limiting process.**

[10 for a; 20 for b]

5. The graph of the function $y = f(t)$ at right consists of two lines and a semicircle.



a. Use the graph to give the exact values of (i) $\int_1^2 f(t)\, dt$ and (ii) $\int_1^4 f(t)\, dt$.

b. Let $g(x) = \int_0^x f(t)\, dt$ for $0 \le x \le 7$. Answer these questions about g:

(i) Give the values of g(0), g(2), g(4), and g(6).

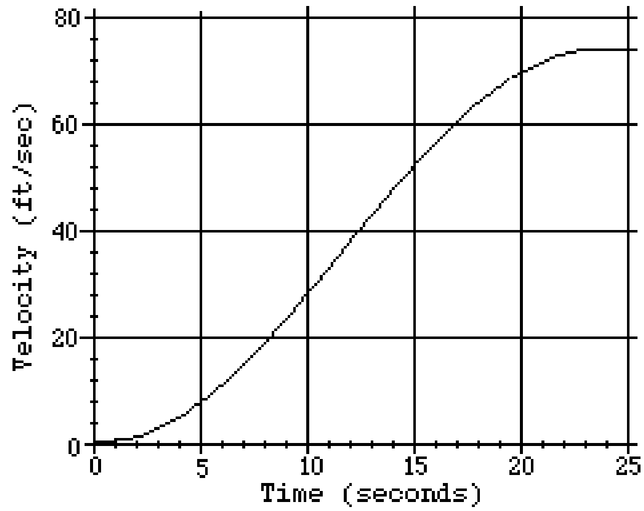
(ii) On what interval(s) is g increasing?

(iii) Where, if at all, does g have a relative maximum? a relative minimum?

(iv) For EXTRA CREDIT, draw a graph of g. (You'll have to draw your own coordinate system.)

[25 pts for a: 5, 5,10; 10 pts for b]

6. The velocity, v(t), of a car pulling onto an interstate is shown in the graph below. To estimate the total distance the car travels during the first 25 seconds, we would find $\int_0^{25} v(t)dt$. We can't find this value exactly, but we can estimate it by dividing the interval [0, 25] into equal subintervals and computing Riemann sums using left endpoints or right endpoints or midpoints of the subintervals. [**Note:** I **know** that area gives total distance in this problem situation. Your job below is to **explain why** area gives distance.]



a. I partitioned [0, 25] into five equal subintervals and computed one of the Riemann sums to be approximately $2 \cdot 5 + 16 \cdot 5 + 40 \cdot 5 + 64 \cdot 5 + 73 \cdot 5$.

(i) Is this a left- or right-hand or midpoint Riemann sum?

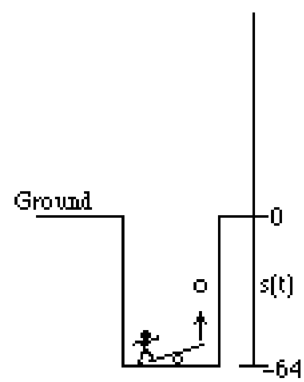
(ii) Draw the rectangles that correspond to this sum on the graph at right.

(iii) Explain why the third term in the sum -- the term $40 \cdot 5$ -- is an estimate for the distance the car travels between times t = 10 and t = 15. (The important thing here is to **explain** why the product $40 \cdot 5$ represents a **distance**.)

b. What, approximately, is the total distance the car traveled during the first 25 seconds? Tell how you got your estimate and answer the question in a sentence.

[a is worth 18 points; b and c are each worth 6 points]

7. A ball is catapulted vertically upward from the bottom of a 64 foot deep hole at the initial velocity of 80 feet per second. With respect to the coordinate line shown (ground level is zero, up is positive) s(t) is the position of the ball t seconds after it begins its flight.



a. We're given that the initial velocity is 80 feet per second, the initial position is 64 feet below ground level. We also know that the acceleration due to gravity is a constant 32 feet per second per second. (i) State these **initial conditions** in terms of the function s and its derivatives. (ii) Then use antidifferentiation to **derive an equation** for s(t). (Obviously you have to show your work. Also, you cannot start with an equation for the position that you may have memorized. It's that equation that you are to derive.)

Note: If you cannot derive the equation, ask me to give it to you so you can do (b) and (c).

b. When does the ball reach its apex? How high is it then?

c. When is the ball at ground level? What is the velocity of the ball then?

The remaining problems are for EXTRA CREDIT.

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[30 possible]

8a. By differentiating, show that $\frac{d}{dx}\left(\arctan\left(\frac{x}{a}\right)\right) = \frac{a}{a^2 + x^2}$. (Obviously, you must show your work.)

Then find $\int_2^{2\sqrt{3}} \frac{2}{4 + x^2}\, dx$ in two ways:

b. Approximately, using your calculator program. (Approximate to two decimal places.)

c. Exactly, using the Fundamental Theorem of Calculus.

Be sure to show what you're doing in both cases.

[20 possible]

9. The graph of a certain function f has the property that the slope of the tangent line at each point (x, y) on the graph is $4x^3 - 5$. The graph also contains the point (1, 2). Find the equation for the function f.

[30 possible]

10. Use your graphing utility to draw the graph of $f(x) = \cos x^2$ on the interval $\left[0, \sqrt{\frac{3\pi}{2}}\right]$. Reproduce your graph on your answer sheet. Note that the graph intersects the x-axis at $\sqrt{\frac{\pi}{2}}$ and at $\sqrt{\frac{3\pi}{2}}$. In answering the following questions, be sure to explain what you did to get the results.

a. Estimate the <u>area</u> of the region between f(x) and the x-axis from 0 to $\sqrt{\frac{\pi}{2}}$.

b. Estimate the <u>area</u> of the region between f(x) and the x-axis from $\sqrt{\frac{\pi}{2}}$ to $\sqrt{\frac{3\pi}{2}}$.

c. Approximate $\int_{\sqrt{\frac{\pi}{2}}}^{\sqrt{\frac{3\pi}{2}}} f(x)\, dx$. d. Approximate $\int_0^{\sqrt{\frac{3\pi}{2}}} f(x)\, dx$.

[30 possible]

11. Here is a "reduction formula" given in the table of integrals: $\int x^n e^x dx = x^n e^x - n \cdot \int x^{n-1} e^x dx$.

(a) Show by differentiating that this formula is correct.

(b) Use the formula to find $\int x^2 e^x dx$.

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GENERAL DIRECTIONS:

Do your work and give your answers on the paper I've provided. (You may do the drawing for Problem 6 on the test paper.) **Answer all problems in sentences, even those requiring a numerical result**. Please remember that a sentence could be as simple as $\int 3x^2\, dx = x^3 + C$. <u>**Show your work and explain what you are doing**</u>. (Your explanations will be graded. "Answers" without explanations are not acceptable.) An **exact value** usually involves a radical or π or e or ln. It is <u>not</u> a decimal approximation given by your calculator.

Note about EXTRA CREDIT: As you know by now, EC points are not nearly as important in determining your test grade as are the points on the required problems. You should therefore always spend time making sure you have the required problems as correct as you can make them. On this test, it is possible to earn up to 110 EC points. Because of this, if you think you have not done well on some of the required problems, it would be worth while to use the full two hours to get some of the EC problems done.