MA-161 (F,06)

Test 5 (Integration By Substitution & Applications)

180 possible points. Name____________________________

DIRECTIONS: Do your work and give your answers on the paper I've provided. **Answer all problems in sentences, even those requiring a numerical result**. **Show your work. Include on your answer sheets pictures of the rectangles, disks, washers, etc. that you use in setting up your integral. If you can't use the FTC to evaluate a definite integral, you may approximate the integral for less credit. You must indicate that you're doing so.**

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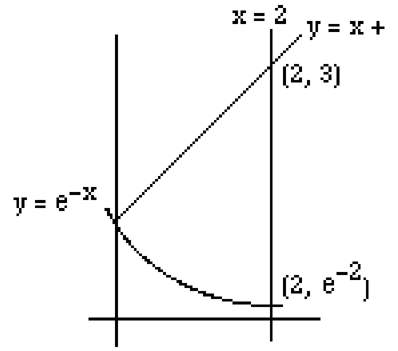
[40 possible: Each part of the question is worth 20 points.]

1. R is the region in the x-y plane that is bounded by the curve $y = e^{-x}$, the line $y = x + 1$, and the vertical line $x = 2$.

a. Find the exact area of the region R. [Reproduce the graph on your answer sheet. Be sure to remember to draw in a rectangle with its height and width indicated.]

b. Find the exact volume of the solid obtained when the region R is revolved about the x-axis. [Again, you have to draw the appropriate picture on your answer sheet. Also remember that $(b^r)^s = b^{rs}$.]

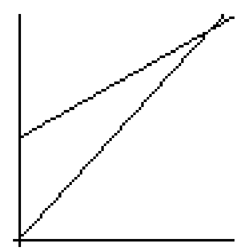


[40: Each part of the question is worth 20 points.]

2. Draw the region, R, that is bounded by the lines $3x – y = 0$, $x + y = 4$, and the x–axis. (Note: the boundary is the x-axis -- not the y-axis.) **(a)** Use integrals to express the area of the region R. (**b)** Write an integral that gives the volume of the solid obtained when the region R is revolved about the y-axis. [You do not have to evaluate any of the integrals.]

[30 points.]

3. A picture of the region bounded by the lines $y = x + 2$, $y = 2x$ and the y-axis is shown at right. A solid has as its base the region shown. Every cross section of this solid by a plane perpendicular to the x–axis is a rectangle whose height is twice its base. What is the exact volume of this solid? [The lines intersect at the point (2, 4).]



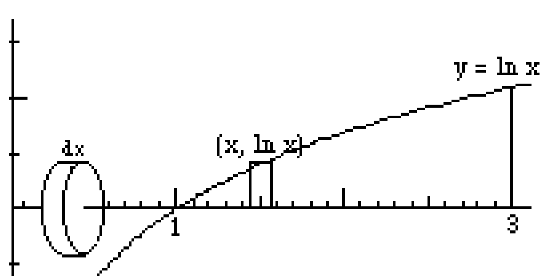
[30 points: 10 points each..]

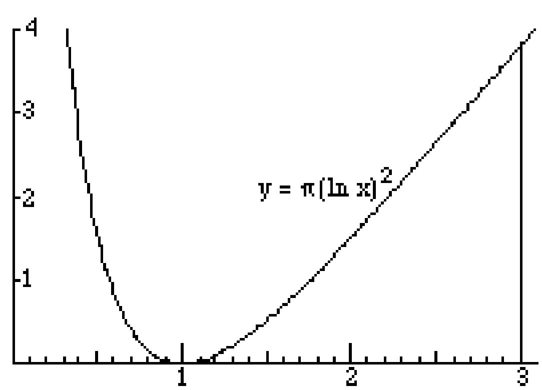
4a. Write an integral expression for the volume of the solid obtained when region bounded by $y = \ln x$, the x–axis, and the line $x = 3$ is revolved about the x-axis?

b. Explain carefully how the question in (a) is related to the following question:

What is the area of the region above the interval [1,3] and below the graph of $y = \pi(\ln x)^2$?

c. Even though it's unlikely that you can find an antiderivative to compute the integral in (a), you can still give an approximation for the volume of the solid in (a). Approximate the volume correct to the first decimal place. Tell what you did to get your value.





[20: 5 each]
5. Find these. Rewrite the given integral and show your work, using correct notation. Simplify all results.

a. $\int x \cdot e^{(-1/2)x^2} dx$ b. $\int \frac{(\ln x)^2}{x} dx$ c. $\int_0^{\pi/2} (\sin x + 1)^3 \cos x \, dx$ d. $\int \frac{3x}{\sqrt{x^2 - 1}} dx$

[20: 15, 5]
6. Evaluate the following definite integral by substitution, using the method of changing the limits of integration, and letting $u = 25 - x^2$ Show all your steps logically using correct notation. **After you've found the value, describe briefly at least one way that you could use to determine whether the numerical value you got for the definite integral is close or not. Then do it.** The integral to evaluate is $\int_3^4 x\sqrt{25 - x^2}\, dx$. [Note: If you can't do this one, you may do this problem: $\int_0^1 2x(1 + x^2)^3 dx$, letting $u = 1 + x^2$. This would be for less credit.]

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The rest of the problems are EXTRA CREDIT. Do them only after you've completed, and checked over, everything else you can do on the test. [All problems are worth 30 points each.]

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7. In Problem 2a, evaluate the definite integral to find the area of the region R. Then show that your result is correct by finding the area using the formula for the area of a triangle.

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8. Derive the formula for the area of the ellipse with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

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9. Find the volume of the ellipsoid obtained by revolving the top half of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the x-axis.

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10. The formula for the volume of a right circular cone with height h and base radius r can be obtained when the region bounded by the line $y = \frac{r}{h}x$, the x-axis and the vertical line $x = h$ is revolved about the x-axis. Find this volume. (Obviously, you must draw a figure and show your work.)

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11. **R** is region bounded by the curve $y = 1 / x^2$ on the top, the x-axis on the bottom, and the vertical lines $x = 1$ and $x = 4$ on the left and right. Find the number b such that the vertical line $x = b$ cuts the area of **R** into two equal pieces.

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12. Find these. Rewrite the given integral and show your work, using correct notation. Simplify all results.

a. $\int \theta^{n-1} \sin \theta^n \, d\theta$ b. $\int \sin^n\theta \cos \theta \, d\theta$ $(n \neq -1)$ c. By Substitution, find $\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx$
