170 possible points

MA-331 (W, 07)
Test 2

Name ________________________

The directions are given on the bottom of this page. Read them first. The questions you have to answer are preceded by a **bolded** numeral and/or letter. Put all, and only, Problem **1** on a single page; all, and only Problem **2** on its own page, etc.

****

So far in this course, we have investigated "Absolute Geometry", the geometry obtained by assuming the sets of axioms described below.

The <u>Incidence Axioms</u>: (I-0) Every line is a set of points; (I-1) Two points determine a unique line; and

(I-2) A line contains at least two points; the plane contains at least three non-collinear points.

[35 poss: 5, 20, 10]

The <u>Distance Axioms</u> The two Distance Axioms were these:

D-0. There is a function, d, which associates with each ordered pair of points, (P,Q), a unique real number, d((P,Q)).

D-1. Every line has a coordinate system.

1a) Complete this definition of coordinate system:

Def) A function f with domain the line *l* and range the set **R** of real numbers is a <u>coordinate system for line *l*</u> iff these two conditions hold:

(1) f is a one-to-one correspondence between *l* and the real numbers; and

(2) ... [You give the second condition.]

1b) Suppose you have a line *l* and a coordinate system, f, for *l*. You define a new function, g, with domain *l* and range **R** as follows: for each P on *l* , $g(P) = a \cdot f(P) + b$ where a and b are real numbers with $a \neq 0$.

(i) Prove that g is onto **R**. (If you can't prove that g is onto, you can prove that g is one-to-one for less credit.)

(ii) Tell all the values of a and b would make g a coordinate system for *l*?

1c) We define the concept of betweenness in terms of distance, and then define the basic geometric figures (segment, ray, angle, etc.) in terms of betweenness. Give a careful definitions of

(i) A - B - C (You recall that this is shorthand for "B is between A and C."); and

(ii) the ray $\overrightarrow{AB}$.

Note: Your definition must be logically (and grammatically) complete. For example, if you were asked to define the concept of congruence for segments, you would write:

Given segments $\overline{AB}$ and $\overline{CD}$. Then $\overline{AB}$ $\overline{CD}$ iff AB = CD.

Similarly, the definition for line segment could be this:

Given two points, A and B. Then $\overline{AB}$ = { P | P = A or A - P - B or P = B }.

****

<u>**DIRECTIONS**</u>

Write all your answers on your answer sheet. Be sure to number the pages and the problems. All definitions, axioms, etc. **must be given as complete sentences** unless otherwise specified. Be careful about your **use of notation.**

[15 poss]
The <u>Plane Separation Postulate</u> The PSP says

PSP) Given a line *l*. Then there are sets H_1 and H_2 satisfying

(1) The plane is H_1 *l* H_2;

(2) H_1, *l*, H_2 are pairwise disjoint;

(3) H_1 and H_2 are both convex sets; and

(4) For any points P and Q: if P H_1 and Q H_2, then $\overline{PQ}$ intersects *l*.

2) Use the PSP to give a careful indirect proof of this theorem:

Given a line *l* and a segment $\overline{AB}$. If $\overline{AB}$ intersects *l* at a single point C between A and B, then A and B lie on opposite sides of *l*.

Be sure to tell <u>all</u> the parts of the PSP that you're using in your proof.

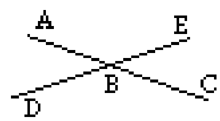
Note: If you can't do **the question above**, you may give a careful definition of "convex set" for partial credit.

[30 poss: 15 each]
The <u>Angle Measure Axioms</u> There were five Angle Measure Axioms.

3a) State **carefully** and **in detail** the Angle Construction Postulate.

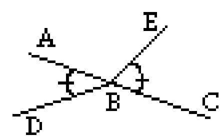
The Vertical Angle Theorem basically says that vertical angles are congruent. In symbols, this says

Given five points A, B, C, D, and E, no four of which are collinear.
If A - B - C and D - B - E, then m   ABD = m   CBE.



3b) Prove this "psuedo converse" of the Vertical Angle Theorem:

Given five points A, B, C, D, and E, no four of which are collinear.
If A - B - C, and D and E are on opposite sides of $\overleftrightarrow{AB}$,
and m   ABD = m   CBE, then D - B - E.



[Hint: You might try the "method of wishful thinking" that you used in proving the psuedo converse of the Angle Supplement Postulate. Also, if you can't prove the theorem in 3b, you can prove the Vertical Angle Theorem for less credit.]

[40 poss: 5, 5, 10]
<u>Side Angle Side</u> The final axiom we've used so far is SAS. SAS was, and will be, used to prove many theorems, among which is the Isosceles Triangle Theorem.

4a) State in words, **(b)** then restate in symbols, and then **(c)** prove, the Isosceles Triangle Theorem.

SAS was also used to prove ASA which in turn was used to prove the converse of the Isosceles Theorem.

4d) State in words, **(e)** restate in symbols, and then **(f)** prove, the <u>Converse</u> of the Isosceles Triangle Theorem.

[30 poss: 10, 20]
The Exterior Angle Theorem (EAT) could be used to prove a number of important theorems.

5a) State EAT in words. Then draw a figure and state EAT symbolically, using the symbols in your figure.

One thing EAT was used to prove was this theorem from your last set of homework problems:

If two lines are cut by a transversal so that alternate interior angles are congruent, then the lines are parallel.

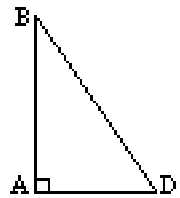
Here is a restatement of this theorem: (It's the same as was given on the problem set.)

Given lines two lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ with transversal $\overleftrightarrow{AC}$ with B and D on opposite sides of $\overleftrightarrow{AC}$.
If $m\angle BAC = m\angle DCA$, then $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

5b) Prove this theorem indirectly using EAT

[20 poss]
6) You proved a number of theorems about Saccheri Quadrilaterals on the last set of homework problems. You may use any of the those theorems plus anything else we've proved in this Unit to prove the following theorem:

Given right triangle ΔABD with right angle at A. Let E be a point on the same side of $\overleftrightarrow{AD}$ as B such that $m\angle ADE = 90$, and let C be the point on $\overrightarrow{DE}$ such that DC = AB. If BC = AD, then the sum of the measures of the angles of ΔABD is 180.



[Hint: The construction gives a Saccheri Quadrilateral, so you can use any of the the theorems you proved for homework to do this proof. Don't take this to mean that you <u>have</u> to use the Saccheri Quad theorems!]
****

The following problems are for EXTRA CREDIT. You may do any or all of them, but <u>only after you've completed everything else you can do on the test</u>. Questions 7 - 10 are worth 15 points each and 11 is worth 30 points. However, it is more important for your test score to do well on the required part of the test.

7. After stating the Incidence Axioms, I asked if it was possible to prove that a line contained at least three points <u>using just the Incidence Axioms</u>. The answer was no. Give in detail an argument supporting this answer.

8. Draw a labeled picture to show that ASS is not a congruence theorem. [You have to say explicitly which sides and angles are congruent.]

9. On the GEOMETRY PROBLEMS handout, I gave a complicated proof for this theorem:

Th) Given a line and a point not on it. Then there is at most one line thru the point perpendicular to the given line.

My proof was complicated because I didn't use EAT but instead essentially incorporated a proof of EAT into my proof. Using EAT, the theorem above is easy to prove <u>indirectly</u>. Draw a labeled figure and give such an indirect proof.

10. Once the theorem in (9) is proved, it is easy to use it to prove indirectly the following theorem:

Th) If two lines are perpendicular to the same line, then they cannot intersect.

Give such a proof.

11. It is possible to prove the theorem in Problem **5b** without using EAT. One way to do it is to drop perpendiculars from the midpoint of $\overline{AC}$ to the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. Then use the theorem in **3b** and the theorem in **10**. Give such a proof.