Logarithms

$\log_2 8$ is a number. What number? $\log_2 8$ is the power that 2 must be raised to to get 8. Thus, $\log_2 8 = x$ means $2^x = 8$. By inspection, $x = 3$. Therefore, $\log_2 8 = 3$.

Definition $\log_b r$ is the power that b must be raised to to get r. I.e., $\log_b r = s$ if, and only if, $b^s = r$

You must be able to translate log statements to exponent statements and exponent statements to log statements.

Examples: $\log_w(uv) = t$ translates to $w^t = uv$
$b^{x+y} = rs$ translates to $\log_b(rs) = x+y$.

Elementary properties of logarithms that come from the definition:

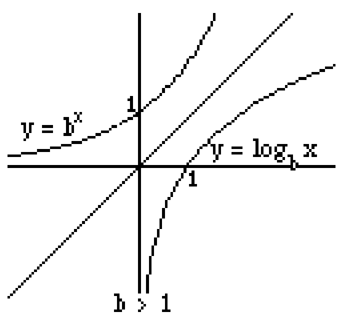
1) $\log_b u = \log_b v$ iff $u = v$.

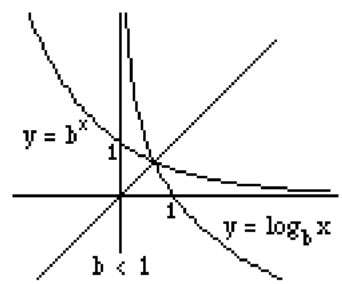
2) $\log_b b = 1$ 3) $\log_b 1 = 0$ 4) $\log_b x$ is undefined if $x \le 0$.

5) $\log_b b^r = r$ 6) $b^{\log_b b^r} = r$

7) If $f(x) = \log_b x$ and $g(x) = b^x$, then $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$.

8) Therefore, $\log_b x$ and b^x are inverses of one another. Consequently, if (r, s) is on the graph of $y = \log_b x$, then (s, r) is on the graph of $y = b^x$, and conversely. Also, the graphs of $y = \log_b x$ and $y = b^x$ are symmetric about the line $y = x$.





9) a. $\log_b(rs) = \log_b r + \log_b s$

b. $\log_b\left(\frac{r}{s}\right) = \log_b r - \log_b s$

c. $\log_b r^s = s\log_b r$

<u>Proof of (a)</u>: Let $x = \log_b r$ and $y = \log_b s$. Translate to exponents to get $b^x = r$ and $b^y = s$. Multiply equals by equals to get $b^x\ b^y = rs$. But $b^x\ b^y = b^{x+y}$ so $b^{x+y} = rs$. Translating this back to logarithms to get $\log_b(rs) = x + y$. Now substitute $\log_b r$ for x and $\log_b s$ for y. Hence, $\log_b(rs) = \log_b r + \log_b s$.

<u>Proof of (c)</u>: Let $x = \log_b r$ so $b^x = r$. Raising both sides to the power s, we get $(b^x)^s = r^s$. But $(b^x)^s = b^{xs}$ so $(b^{xs}) = r^s$. Translating this back to base b logs gives $\log_b r^s = xs$. Since x is $\log_b r$, then by substitution, $\log_b r^s = (\log_b r)s$ or $\log_b r^s = s\log_b r$.

You should be able to prove (b).

Conventions: $\log_b(rs)$ is written $\log_b rs$. If you want to write $\log_b r$ times s, you write $(\log_b r)s$ or, better, $s\log_b r$. Also, $\log_b r + s$ means $(\log_b r) + s$. There are additional conventions about notation that you'll learn as you go along. One thing you have to guard against is making up "rules" that are not really rules. Here are some **wrong** things:

$\log_b(r + s) = \log_b r + \log_b s$. $\log_b\left(\frac{r}{s}\right) = \log_b(r - s)$. $\log_b\left(\frac{r}{s}\right) = \frac{\log_b r}{\log_b s}$.

Until you're very comfortable with the log properties, **think** before you write.

10) Change of Base Formula: $\log_a x = \frac{\log_b x}{\log_b a}$.

Start of a proof of (10) for you to finish: Let $y = \log_a x$ so $a^y = x$. Hence, $\log_b a^y = \log_b x$. [See if you can do the rest.]

11) The two most often used bases are base 10 and base e. $\log_{10}x$ is written log x. These are called common logs. $\log_e x$ is written ln x. These are natural logs.

The best way to read "ln x" is "log to the base *e* of *x*"to remind yourself that ln x is just a base *e* logarithm.

Both common logs and natural logs are on your calculator. If you want to find the values of logs to other bases, use the Change of Base Formula.

Example $\log_2 5 = \frac{\log 5}{\log 2}$ or $\frac{\ln 5}{\ln 2}$. Try these on your calculator. You may want to find $\log_2 8$ just to convince yourself.