Trigonometry Basics

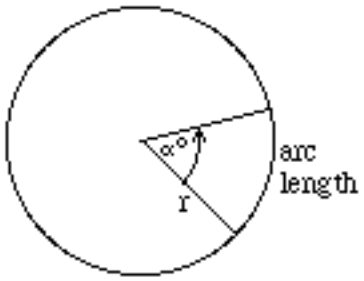
1. Radian measure of angles.

a. Circumference of a circle is $2\pi \cdot$radius so circum of unit circle is 2π.

b. If a central angle of a circle with radius r measures $\alpha°$, then

$$\frac{\text{length of subtended arc}}{2\pi r} = \frac{a}{360}.$$

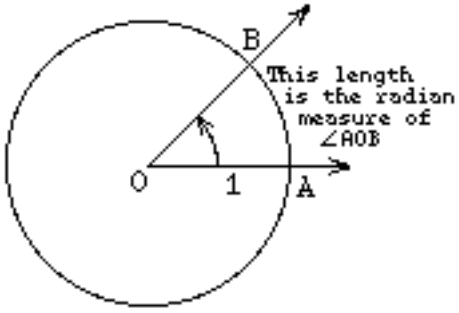
$$\text{arc length} = \frac{a}{360} \cdot 2\pi r$$



c. The radian measure of an angle is defined as folows:

Consider the angle in question as the central angle of a unit circle. Then

$$\left.\begin{matrix}\text{radian measure}\\ \text{of the angle}\end{matrix}\right\} = \left\{\begin{matrix}\text{directed length}\\ \text{of subtended arc}\end{matrix}\right.$$



d. The definition above gives this formula which relates radian and degree measure:

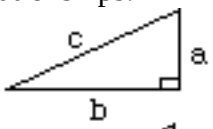
$$\frac{\text{degree measure}}{360} = \frac{\text{radian measure}}{2\pi}$$

e. You must know (i.e., have memorized) the radian measures of the standard angles (i.e., the angles whose degree measures are integer multiples of 30 and 45.)

degs	0	30	45	60	90	120	135	150	180	210	225	etc
rads	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π			etc

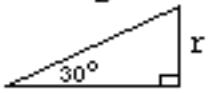
2. Some right triangle relationships:

Pythagoras:

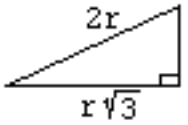


$$c^2 = a^2 + b^2$$

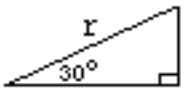
30-60-90 Δ's:



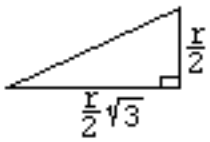
$\Rightarrow$



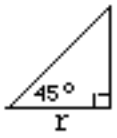
or



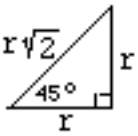
$\Rightarrow$



45-45-90 Δ's:



$\Rightarrow$

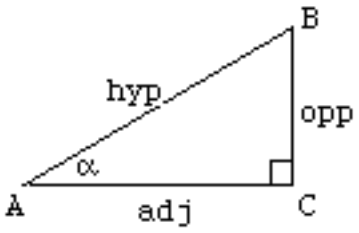


3a. Definition of trigonometric functions for right triangles:

$$\sin\alpha = \frac{\text{opp}}{\text{hyp}} \qquad \csc\alpha = \frac{\text{hyp}}{\text{opp}} = \frac{1}{\sin\alpha}$$

$$\cos\alpha = \frac{\text{adj}}{\text{hyp}} \qquad \sec\alpha = \frac{\text{hyp}}{\text{adj}} = \frac{1}{\cos\alpha}$$

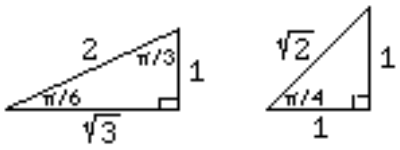
$$\tan\alpha = \frac{\text{opp}}{\text{adj}} = \frac{\sin\alpha}{\cos\alpha} \qquad \cot\alpha = \frac{\text{adj}}{\text{opp}} = \frac{\cos\alpha}{\sin\alpha} = \frac{1}{\tan\alpha}$$



3 (Continued)

b. You must know the values of the trig functions for the standard acute angles:

α	π/6	π/4	π/3
sin α	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$
cos α	$\sqrt{3}/2$	$\sqrt{2}/2$	1/2
tan α	$\sqrt{3}/3$	1	$\sqrt{3}$

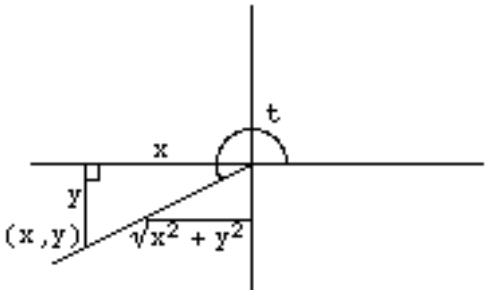


4. Definition of the trig functions for <u>any real number</u>:

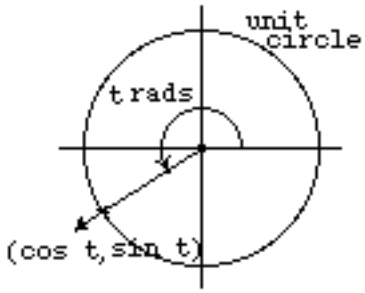
a. For any real number, t, construct an angle in standard position with radian measure t. Choose an arbitrary point (x,y) on the terminal side. Then

$$\sin t = \frac{y}{\sqrt{x^2 + y^2}} \qquad \cos t = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\tan t = \frac{y}{x} \quad \text{etc.}$$



b. By definition, we have the relationship given in the figure at right. The point where the terminal side intersects the <u>unit</u> circle has coordinates (cos t, sin t).



c. To apply the definitions to find values of the trig functions for the standard angles (and others, too) do this: [I'll use 7π/6 as an example.]

-- Draw the angle in standard position.

-- Drop the perpendicular from a point on the terminal to the x-axis. (**Always** to the **x-axis**.)

-- The resulting right triangle will have an acute angle at the origin. Find the acute angle.

-- Assign the appropriate **directed** lengths to the sides of the triangle. (The hypotenuse is always positive.)

-- The values of the trig functions will be the same as the value of the funtion for the right triangle with the appropriate sign.