

[140 possible points]

Problem 1a is on a separate sheet. You'll do it first.

Name _____

[20 possible]

1. Problem 1a is on a separate sheet. Problems 1b and c are on the last page of the test. **Do the drawing on that page but give your work and answers on your answer sheet.**

[10 possible]

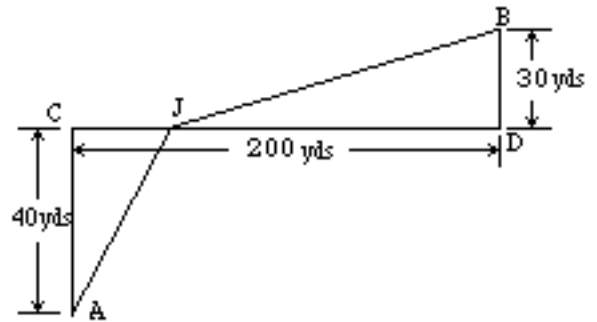
2. Simplify: $\sin^2(\tan^{-1}x)$. [If you can't do this one, you may simplify $\sin(\arccos \frac{a}{b})$ for less credit.]

[10 possible]

3. An open rectangular box with a square base has a volume of 4 cubic feet. **Define variables** and express the surface area of the box as a function of the length of a side of the base.

[30 possible]

4. Two houses, A and B, are located on a rural road as shown in the figure. A is 40 yards from the road; B is 30 yards from the road. The telephone company will put a junction box on the road at a point J and run underground cables from J directly to houses A and B. The house at A wants fiber optic cable that costs \$12 per yard while the B house wants standard cable that costs \$10 per yard.



i) **Define variables** and write the total cost of the project as a function of the distance J is from C.

ii) Choose a **reasonable viewing rectangle** and draw a graph of the function in (i). Reproduce your graph on your answer sheet.

Be sure to **label the axes**. (To save time, I'll tell you that the total cost will be between \$2000 and \$3000.)

iii) How far from C should the junction box be located in order to minimize the total cost of laying the cable? What will the cost be in this case?

[20 possible]

5. A jogger's heart rate (in beats per minute) is related to her speed (in feet per second) by a **linear** equation. Assume the jogger's heart rate is 105 beats per minute at the speed of 6 feet per second and 130 beats per minute at 10 feet per second.

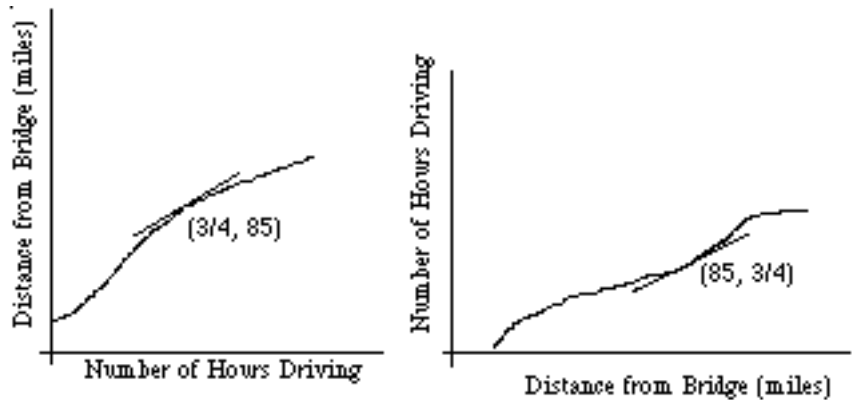
i) **Define variables** and write a linear equation for the jogger's heart rate as a function of her running speed.

ii) Tell the slope of the line. Then tell in detail what the slope means in the context of this problem situation. Be sure to use the correct units of measurement for the slope in your explanation.

[10 possible]

11. Suppose I leave Indian River, a town 20 miles south of the Mackinaw Bridge, and drive south on I-75. The graph below left might be the graph of the distance I am from the bridge as a function of the number of hours I've been driving.

The inverse of this function interchanges all the ordered pairs on the graph. The graph of the inverse function might be like the picture below right. Suppose I estimate the slope of the tangent to the left graph at the point $(3/4, 85)$ to be 56. I also estimate the slope of the tangent to the right graph to be .02.



Explain in detail what each of the two slopes means in the context of this situation, using the appropriate units of measurement for each.

[25 possible]

12. On the coordinate system below, draw an accurate graph of the function defined by the equation $y = 2^x$. Then draw an accurate graph of the inverse of the function defined by this equation. (You want the **inverse function**, not the reciprocal.) [Do all of this problem on this page below the graph.]

(a) What is an equation for the inverse function? Give your answer in the form $y = \underline{\hspace{2cm}}$.

(b) At the point $(0,1)$ on the graph of $y = 2^x$, draw carefully the tangent line and estimate its slope. Show how you're estimating the slope.

(c) At the point $(1,0)$ on the graph of the inverse function, draw the tangent line and estimate its slope. Again, show your work.

(d) **EXTRA CREDIT** What do you suspect is the relationship between the slopes of the two tangent lines you drew in (b) and (c). [Remember, you write in sentences!]

(e) **More EC** You know, I hope, that the point $(0,1)$ is on the graph of $f(x) = 2^x$ and that $(1,0)$ is the corresponding point on the graph of $y = f^{-1}(x)$. What do you think the relationship is between the slopes of the tangents to any pair of corresponding points of this nature on the two graphs? Answer in a paragraph on your answer sheet. You might want to draw on that sheet graphs of the two functions to illustrate your exposition.

