MA-161 (F,07)
Test 2 (Derivatives)

[180 possible points.] Name_____

DIRECTIONS and FORMULAS: On another sheet.

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[25 possible: 5 each]

1. Find the derivative with respect to x for each of these functions. a is a positive constant. Simplify your result. [Pay close attention to the last function.]

(a) $f(x) = 4 - \frac{1}{3}x^2$; (b) $g(x) = \sqrt{a^2 - x^2}$; (c) $y = x^3 \cdot \ln x$.; (d) $t(x) = \tan^{-1}\sqrt{x}$; (e) $y = (\sin x)^{3x}$

[20 possible]

2. Using one of the two forms of the definition of derivative, prove $\frac{d}{dx}(f(x) - g(x)) = \frac{d}{dx}(f(x)) - \frac{d}{dx}(g(x))$.

Note: If you can't prove this rule, prove the constant multiple rule for less credit. (A correct symbolic statement of the constant multiple rule is itself worth 5 points.)

[30 possible]

3. Use one of the two forms of the definition of derivative to derive the formula for either $\frac{d}{dx}(x^n)$ (where n is a natural number), or for $\frac{d}{dx}(\cos x)$.
You must show your work in detail. Please note that you must use the definition of derivative and not logarithmic differentiation to do the derivation. See the formula sheet for useful limits.

Note: If you can't derive the formula for either of the derivatives above, you may derive the formula for $\frac{d}{dx}(e^x)$ for less credit. If you can't do any of these, you may derive the formula for $\frac{d}{dx}(x^2)$ for still less credit.

[40 possible: 10, 10, 20]

4. With the reminders given here, obtain formulas for these derivatives. Obviously you must show your work.

(a) Recalling that $\csc x = \frac{1}{\sin x}$, find $\frac{d}{dx}(\csc x)$. [This is not a question involving the definition of derivative.]

(b) Use the formula $D_x(\ln x) = \frac{1}{x}$ and the Change of Base formula (on the formula sheet) to find y' when $y = \log_b x$.

(c) Use the method for finding the derivative of an inverse function using implicit differentiation (see the formula sheet) to derive the fromula for $\frac{dy}{dx}$ when y = arccos x . **Note**: If you can't do arccos x, you may, for less credit, derive the formula for the derivative of y = ln x using the same procedure.

[20 possible]

5. On the coordinate system I've provided, draw an accurate graph of $f(x) = 4 - \frac{1}{3}x^2$ for $-4 \le x \le 4$. Find the exact (not estimated) slope of the tangent line to the graph where x = 2. Give the (exact) equation of the tangent line. Draw carefully the tangent line to the graph at the point where x = 2.

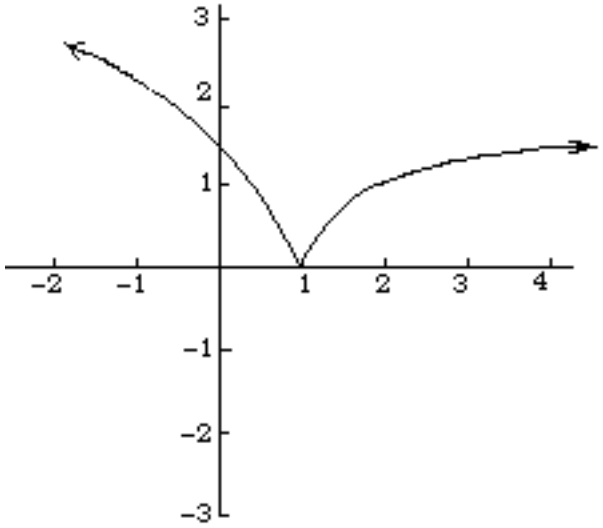
[25 possible]

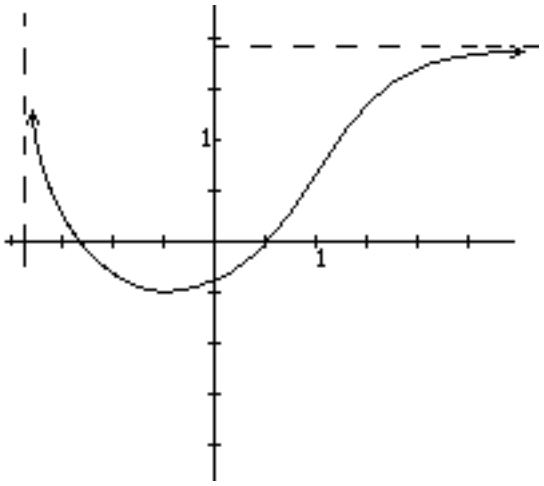
6a). Evaluate exactly $\left.\frac{d^2y}{dx^2}\right|_{x=\,2}$ when $y = e^{\frac{1}{2}x^2}$. (b) Find and simplify $\frac{d^3}{dx^3}(e^x \cdot \sin x)$.

(c) If $f(x) = e^{3x}$, find a formula for $f^{(n)}(x)$. Show how you got your formula. (Note: $f^{(n)}(x)$ means $\frac{d^n}{dx^n}(f(x))$.)

[20 possible: 10 each]

7. Below are the graphs of two functions. On the same coordinate systems, draw approximate graphs of the derivatives of the these functions. Be sure your sketches are consistent with the important features of the original functions.





The following problems are for EXTRA CREDIT. However, do not work on these problems until you have done all you can do, and have checked, problems 1 – 7.

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8. In Problem 6c, you obtained a formula for $\frac{d^n}{dx^n}(e^{3x})$. Now give a formula for $\frac{d^n}{dx^n}(e^{-3x})$.

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9. Give an example of a function, f, and a number, a, having the properties f is defined at a, f is continuous at a, f has a tangent line at (a, f(a)), but f is not differentiable at a.

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10. Differentiate and simplify: (k is a constant. Also, the derivative of arcsine is on the formula sheet.)

(a) $D_x\left(\arcsin\frac{x}{k}\right)$ (b) $D_x\left(\frac{1}{2k}\ln\left(\frac{x-k}{x+k}\right)\right)$ (c) $t(x) = \frac{1}{n^2}x^n(n \ln x - 1)$ where n is a constant.

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11. Differentiate each of the following functions: (a) $y = x^3$; (b) $y = 3^x$; (c) $y = (3x)^{3x}$

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12. Find the equation of the line tangent to the graph of $y = e^{-\frac{1}{2}x^2}$ at the point where x = 1.

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13. Find the points on the graph of $y = \frac{x}{x-1}$ where the tangent line is perpendicular to the line $y = 4x - 1$.
