MA-161 (F,07)
Test 3
Applications of the Derivative

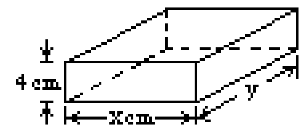
[155 possible points] Name ________________________________

The instructions are on another sheet. **Read them first**.

****

[35 possible]

1. You are to design a rectangular box with a top whose volume is to be 400 cm^3. Its height must be 4 cm. (See the figure at right.) The ultimate questions are these: (i) What are the dimensions of the box with the smallest possible surface area, and (ii) what is that minimal surface area?



Let x be the length of one of the sides of the box as shown in the figure, and let S be the total surface area (sides, top and bottom) of the box.

a. Using the fact that the volume is 400 cm^3, write the length, y, of the remaining side as a function of x.

b. Write the surface area, S, as a function of x. **Note:** If you cannot get an equation for S, **ask me** and I'll give it to you for 5 points off. At least you'll be able to do the rest of the problem which is worth almost 90% of the points.

c. Use the methods of calculus to find the value of x that makes S a minimum.

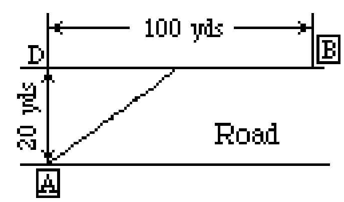
d. Use the Second Derivative Test to show that the value you got does minimize the area. [Show all your work so I can tell exactly what you're doing. Show explicitly your use of the Second Derivative Test. This requires words.]

e. Answer the questions in the original problem.

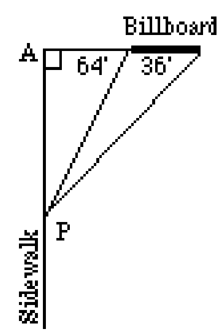
[35 possible]

2. Choose one of these two problems. The first contains some parameters so is worth full credit. The second is less general so is worth 90% credit.

a. Fiber optic cable is to be run between two office buildings, A and B, that are separated by a 20 yard roadway as shown at right. The cable must start at point A, pass under the roadway, and then continue under the ground to the point B. It costs r dollars per yard to bury the cable under the roadway, and it costs s dollars per yard to continue the cable above ground to point B. (Asume $r > s$.) Point D is the point directly across the roadway from A, and it is 100 yards from D to B. Find where the cable should emerge from under the road in order to make the total cost as small as possible. (You do not have to check to see that result gives the minimal cost.)



b. A person, P, is walking along a straight sidewalk looking at a billboard. The billboard is 36 feet wide, it is perpendicular to the sidewalk, and its nearest edge is 64 feet from A, the point where the person will pass the billboard. How far from A will she be when she has the best (i.e., largest) viewing angle of the billboard?

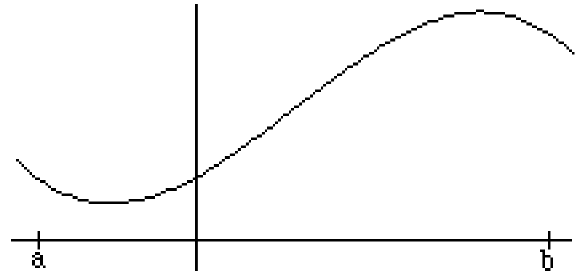


[30 possible]

3. Use the methods of calculus to do the graphing analysis for the function given below. All answers must be exact; i.e., no decimal approximations that you just read from your calculator. You may use decimal approximations when you draw he graph. **Absolutely do not use your calculator to draw the graph**.

Let $f(x) = \frac{1}{9}x^4 - 2x^2 + 7$. Find the relative extrema and inflection points. Use the Second Derivative Test where relevant, stating what your conclusion is. Tell the intervals on which f is increasing and on which it is decreasing. Tell where the graph is concave up and where it's concave down. **On the coordinate system I've provided**, draw the graph of f, labeling the extrema and inflection points on the graph. Write the graphing summary on that page as well. Your graph should be for –4 £ x £ 4 (but the domain is the set of all real numbers.)

[25 possible: 18 for a; 7 for b]
4a. Complete this statement of the Mean Value Theorem (MVT) in the space below. Draw lines in the figure at right to illustrate its geometric meaning, labeling the appropriate points. [You may complete the statement and draw the figure on this paper.]



MVT) Given a function f which is defined on the closed interval [a, b]. If ...

b. On the answer sheet, draw a picture of the conditions given in the theorem below. Then use the MVT to prove the theorem.

Theorem) Assume the function f is continuous on the interval [r, s] and is differentiable on (r, s).
If f(r) = f(s), then there is a number t in (r, s) such that f'(t) = 0.

[30 possible: 20 for a; 10 for b; 5 EC points for c]
5. A ladder 16 feet long leans against a building. The bottom of the ladder slides away from the building at the constant rate of 2 feet per second. (a) How fast is the top of the ladder sliding down the wall when the bottom of the ladder is 10 feet from the base of the building? (b) How fast is the angle between the top of the ladder an the building changing when the bottom of the ladder is 10 feet from the base of the building? (c) How fast is the top of the ladder moving when it hits the ground? [Be careful with notation and with signs!]

The following problems are for EXTRA CREDIT. Do not work on these problems until you have done all you can do, and have checked over, problems 1 – 5.

****

6a. Draw a small portion of the graph of $y = f(x)$ near the point $(a, f(a))$ in the first quadrant subject to these conditions: $f'(a) = 0$ and $f''(a) < 0$. <u>You must include the tangent line in your graph</u>.

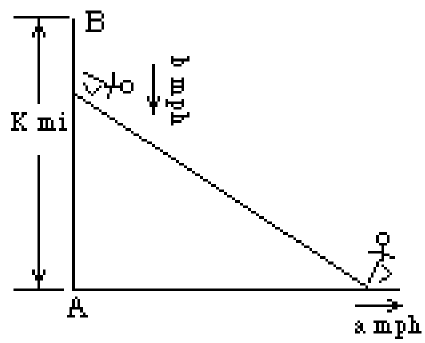
b. Draw a portion of the graph of $y = f(x)$ near the point $(2, f(2))$ given that f satisfies these conditions: $f(2) = 3$, $f'(2) > 0$, $f''(2) = 0$, $f''(x) > 0$ for $x < 2$, and $f''(x) < 0$ for $x > 2$. <u>You must include the tangent line in your graph</u>.

---

7. Assume that a spherical snowball melts in such a way that its radius decreases at a constant rate. Suppose it begins as a sphere of radius 12 cm and it takes 3 hours to disappear. At what rate is the volume changing in one hour? Be sure to define variables and answer in a sentence with appropriate units of measurement. [Reminder: The volume of a sphere is $(4/3)\cdot\pi\cdot(\text{radius})^3$.]

---

8. Town B is K miles directly north of Town A. Alice leaves Town A at noon running directly east at a constant speed of a miles per hour. Also at noon, Betty leaves Town B running directly south toward Town A at the constant speed of b miles per hour. How many hours after noon will the two women be nearest to one-another?

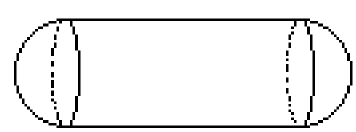


---

9. What are the dimensions of the rectangle with largest area that can be inscribed in the ellipse with equation $\frac{x^2}{r^2} + \frac{y^2}{s^2} = 1$.

---

10. Think about a cylindrical propane tank with hemispherical ends like the one shown at right. You want to construct such a tank to have a volume of 500 cubic feet. However, you want the surface area to be as small as possible. What should the dimensions of the tank be to give the smallest possible surface area? What would the tank look like?



Note: Volume of sphere = $(4/3)\pi(\text{radius})^3$ Surface area of sphere = $4\pi(\text{radius})^2$

Volume of cylinder = $\pi(\text{radius})^2\cdot\text{height}$ Lateral surface area of cylinder = $2\pi(\text{radius})\text{height}$
