MA-161 (F,07)
Test 4 (Integration and FTC)

. Name ____________________________

[20 pts for the first question; 10 pts for the second]

3. Compute $\int_0^w 2x\,dx$ in two different ways, first by using the **definition of definite integral**, and, second, by using the **Fundamental Theorem of Calculus (Part 2).** Tell which is which.

[30 possible]

4. (If you understand the FTC, and you know an antiderivative of cos x, part (c) of this question should take no more than a few minutes.)

Draw a <u>large</u> graph of $f(x) = \cos x$ for $x > 0$ and consider the interval $[0, \pi/3]$.

Suppose we partition $[0, \pi/3]$ into n equal subintervals, each of length $(\pi/3 - 0)/n = \pi/3n$. We then use right-hand endpoints to form the n^{th} Riemann sum: $R_n = \sum_{i=1}^{n} \frac{\pi}{3n}\cos\left(0 + i\cdot\frac{\pi}{3n}\right)$.

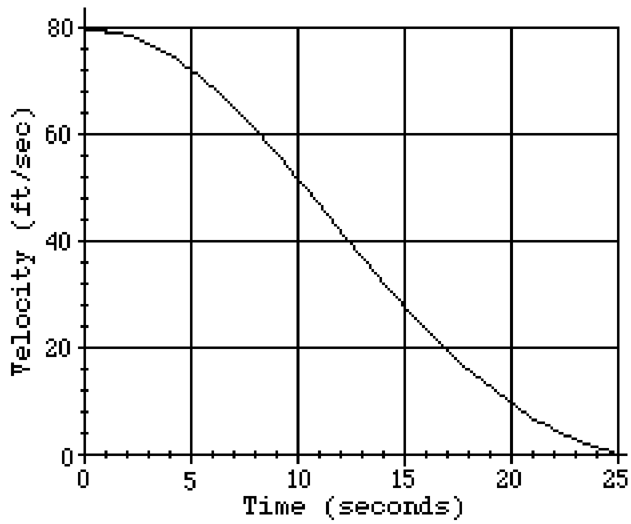
a. On your graph, draw a picture of the situation when n = 2. Then tell <u>exactly</u> the value of R_2.

b. You have the means to approximate R_{20}. Find this value rounded to the third decimal place. Tell how you got your value.

c. **[This is the important part of this problem.]** Finally, we consider $\lim_{n\to\infty} R_n$. What is the <u>exact</u> value of this limit? <u>Tell how you can find the value **without actually having to go through the limiting process**</u>.

[30 possible]

5. The velocity, v(t), of a car that is gently applying its brakes is shown in the graph at right. To estimate the total distance the car travels in coming to rest, we would find $\int_0^{25} v(t)dt$. We can't find this value exactly, but we can estimate it by dividing the interval [0, 25] into equal subintervals and using Riemann sums.



a. Suppose I partitioned [0, 25] into five equal subintervals and computed a Riemann sum to be approximately
$5\cdot80 + 5\cdot72 + 5\cdot54 + 5\cdot28 + 5\cdot10$.

i) Is this a left- or right-hand Riemann sum?

ii) If I were to compute the value of the sum, would it be an over- or an under-estimate for the actual distant the car travels?

b. <u>You</u> estimate the total distance the car travels in coming to rest by dividing the interval [0, 25] into equal subintervals and computing Riemann sums using <u>midpoints</u> of the subintervals. Draw the rectangles you're using on the graph above. Approximately what <u>is</u> the distance the car travels? [Answer in a sentence.]

c. The third term in the sum you computed to answer (b) should have been about $5\cdot40$. Explain why this product is an estimate for the <u>distance</u> the car travels between times t = 10 and t = 15. (<u>The important thing here is to **explain why** the product $5\cdot40$ represents a **distance**.</u>)

d. Then explain why the sum you computed in (b) is an approximation for the total distance the car travels over the interval [0, 25].

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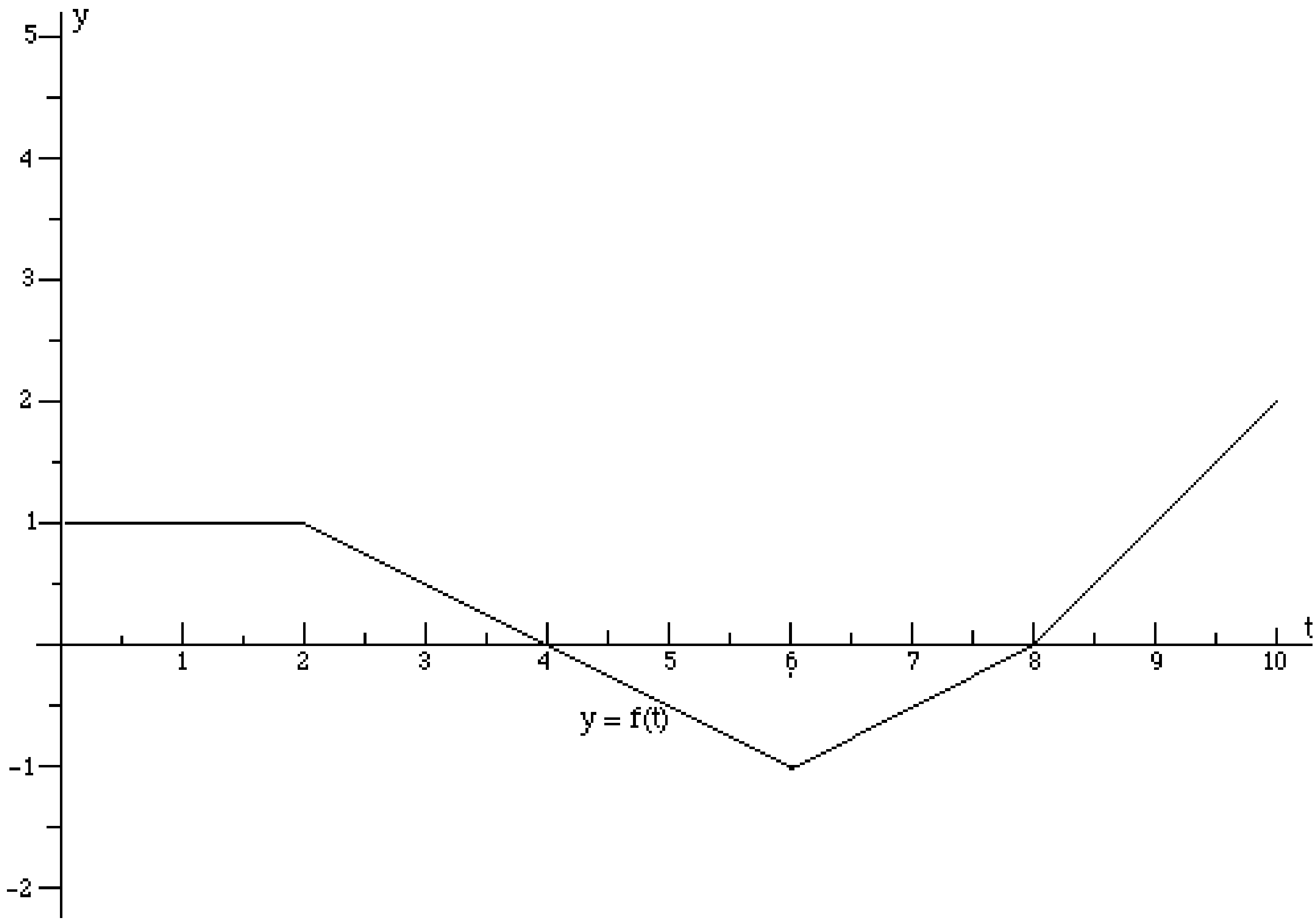
6. Below is the graph of $y = f(t)$. We define a new function, g, by the equation $g(x) = \int_0^x f(t)\,dt$. Answer these questions about the function g. You may answer these questions on this page below the line.

a) Give the values of g(1), g(2), g(4) and g(6). b) On what interval(s) is g decreasing?

c) Give the relative extrema of g. d) Where, if at all, does g have an inflection point?

e) Draw a relatively accurate graph of g on the same coordinate system below as that of f.



The remaining problems are for EXTRA CREDIT. Do them only after you've completed everything you can do on Problems 1 - 6.

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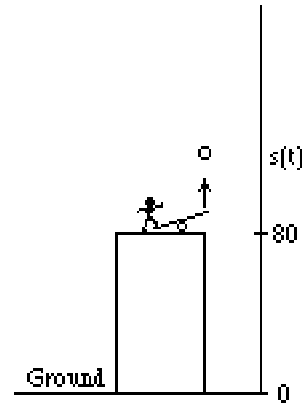
[30 possible]

7. A ball is catapulted vertically upward from the top of a 80 foot high platform at the initial velocity of 64 feet per second. With respect to the coordinate line shown (ground level is zero, up is positive), s(t) is the position of the ball t seconds after it begins its flight.



a. We're given that the initial velocity is 64 feet per second and the initial position is 80 feet above ground level. We also know that the acceleration due to gravity is a constant 32 feet per second per second. State these initial conditions in terms of the functions s, s', and s". Then use antidifferentiation to <u>derive</u> the equation for s(t). (Obviously you have to show your work. Also, you cannot <u>start</u> with an equation for the position that you may have memorized. It's <u>that</u> equation that you are to derive.)

b. When does the ball reach its apex? How high is it then?

c. When is the ball at ground level? What is the velocity of the ball then?

d. What is the displacement of the ball between t = 0 and t = 4 seconds? How far has the ball traveled during that time?

[20 possible]

8. The graph of a certain function f has the property that the slope of the tangent line at each point (x, y) on the graph is $8x^3 - 2x$. The graph also contains the point (1, 13). Find the equation for the function f.

[40 possible: 10,10,10,5,5]

9. Find these. Rewrite the given integral and show your work, using correct notation. Simplify all results.

a. $\int \frac{(\ln x)^3}{x}\,dx$ b. $\int \sin^n q \cos q \, dq$ $(n \pi -1)$ c. $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$ d. $\int_0^p \sin x\,dx$ e. $\int_0^1 \frac{2}{1+x^2}\,dx$

[20 possible]

10. Use your graphing utility to draw the graph of $f(x) = \cos x^2$ on the interval $\left[0, \sqrt{\frac{3p}{2}}\right]$. Reproduce your graph on your answer sheet. Note that the graph intersects the x-axis at $\sqrt{\frac{p}{2}}$ and at $\sqrt{\frac{3p}{2}}$. In answering the following questions, be sure to explain what you did to get the results.

a. Estimate the <u>area</u> of the region between f(x) and the x-axis from 0 to $\sqrt{\frac{p}{2}}$.

b. Estimate the <u>area</u> of the region between f(x) and the x-axis from $\sqrt{\frac{p}{2}}$ to $\sqrt{\frac{3p}{2}}$.

c. Approximate $\int_{\sqrt{\frac{p}{2}}}^{\sqrt{\frac{3p}{2}}} f(x)\,dx$. d. Approximate $\int_0^{\sqrt{\frac{3p}{2}}} f(x)\,dx$.

[30 possible]

11. A *logistic function* is a function of the form $Q(t) = \frac{B}{1 + A \cdot e^{-kt}}$ where A and B are non-zero constants and k is a positive constant. To find an antiderivative, one could first multiply the numerator and denominator by e^{kt}. Using this suggestion, find $\int Q(t)\,dt$.

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