MA-161 (F,07)
Test 5 (Applications of Integration)

180 possible; 30 for each problem. Name___________________________

DIRECTIONS: Do your work and give your answers on the paper I've provided. **Answer all problems in sentences, even those requiring a numerical result**. **Show your work. Include on your answer sheets pictures of the rectangles, disks, washers, etc. that you use in setting up your integral. If you can't use the FTC to evaluate a definite integral, you may approximate the integral for less credit. You must indicate that you're doing so.**

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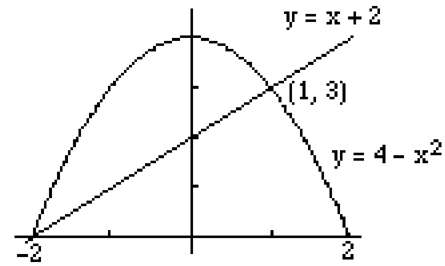
1. Find the exact area of the region R **above** the line $y = x + 2$ and below the parabola $y = 4 - x^2$ shown at right. [Reproduce the graph on your answer sheet. Be sure to remember to draw in a rectangle with its height and width indicated.]



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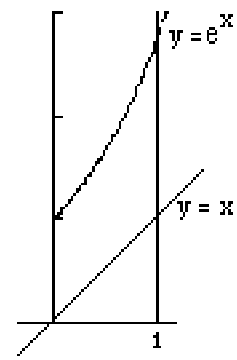
2. Find the exact volume of the solid obtained when the region bounded by $y = e^x$, the line $y = x$, the y–axis, and the vertical line $x = 1$ is revolved about the x–axis. [Remember that $(e^x)^n = e^{nx}$.]



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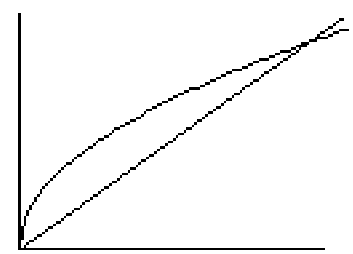
3. A picture of the region bounded by the lines $y^2 = x$ and $2y = x$ is shown at right. This region is revolved around the **y-axis**. Write an integral expression for the exact volume of the resulting solid? [You do not have to evaluate the integral



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4. Draw the region that is bounded by the lines $3x - y = 0$, $2x + y = 10$, and the x–axis. Then use integrals to express the area of the region. [You do not have to evaluate the integral.]

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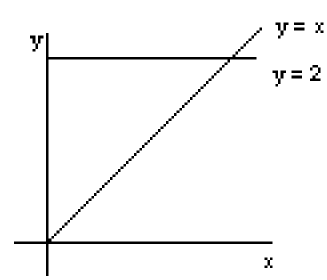
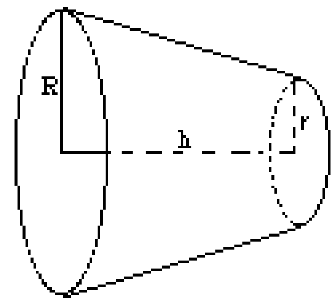
5. The figure at right is the region bounded by the lines $y = x$, $y = 2$, and the y-axis. The lines intersect at (2, 2). A solid has as its base the region shown. Every cross section of this solid with a plane perpendicular to the x-axis is a rectangle whose height is half its base. Write an integral expression for the exact volume of this solid? [You do not have to evaluate the integral.]



6. Choose <u>one</u> of these two problems:

A. Use calculus to derive a formula for the frustum of the right circular cone with the dimensions shown in the figure at right.



B. Use calculus to derive a formula for the solid obtained when a semi-ellipse of the ellipse with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is revolved about the x-axis. Then, using what you know about the volume of a sphere, argue that your formula still works in the case when a = b = r.

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The rest of the problems are EXTRA CREDIT. Do them only after you've completed, and checked over, everything else you can do on the test. [Problems 7 is worth 40 points. The others are worth 30 points.]
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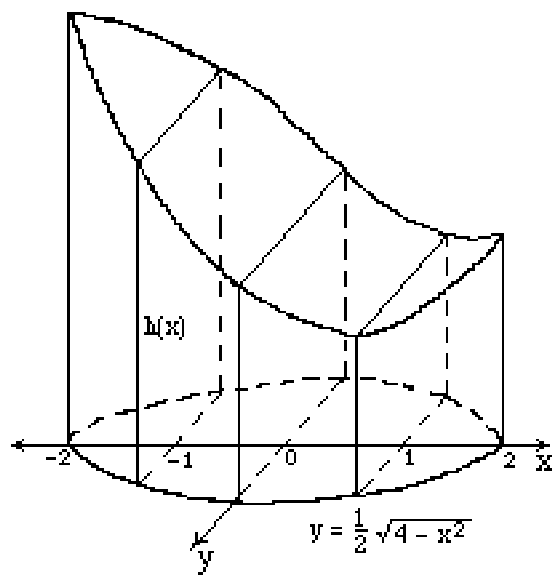
7. The solid at right has an elliptical base with equation $\frac{x^2}{4} + \frac{y^2}{1} = 1$. Cross sections perpendicular to the x-axis are rectangles whose heights, h(x), are an unknown function of x. Measurements of the height were made at one centimeter intervals (all measurements are in centimeters) and are given in the table below.

Values of x	2.0	1.0	0.0	–1.0	–2.0
Heights h(x)	2.0	1.0	3.0	4.0	4.5



a. Write the volume of the solid as a <u>definite integral</u>. Your integrand will contain h(x).

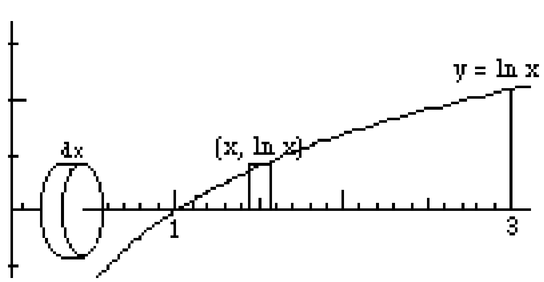
b. Using methods as on the volume problem given in class, approximate the volume of the solid using trapezoids. You must show all your work and explain what you're doing.

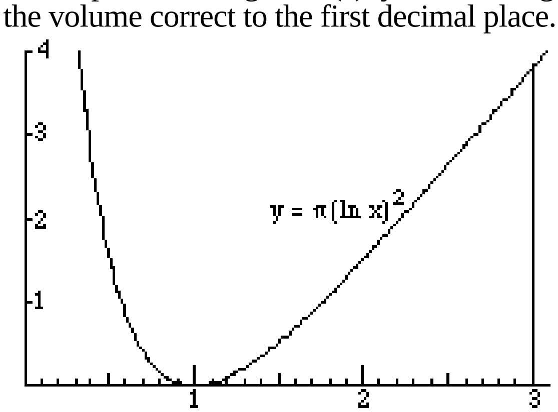
8a. Write an integral expression for the volume of the solid obtained when region bounded by y = ln x, the x–axis, and the line x = 3 is revolved about the x-axis?

b. Explain carefully how the question in (a) is related to the following question:

What is the area of the region above the interval [1, 3] and below the graph of $y = p(\ln x)^2$?

c. Even though it's unlikely that you can find an antiderivative to compute the integral in (a), you can still give an approximation for the volume of the solid in (a). Approximate the volume correct to the first decimal place. Tell what you did to get your value.





[More EXTRA CREDIT problems.]

9. Write an integral expression for the area enclosed by the ellipse with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Then evaluate the definite integral by interpreting it as the area of a familiar figure.

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10. Find the area of the region described in Problem 4 by evaluating integrals. Then show your result is correct by finding the area using the formula for the area of a triangle.

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11. Set up an integral expression for the volume of the solid obtained when the region R in Problem 1 is revolved about the horizontal line $y = -1$. (You don't have to simplify nor evaluate the intergral.)

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12. The region between the curves in Problem 3 is the base of a solid. Cross sections perpendicular to the y-axis are semicircles. Write an integral expression for the volume of the solid. (You don't have to simplify nor evaluate the intergral.)

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14. The formula for the volume of a sphere with radius r can be obtained by by finding the volume of the solid obtained when the region between the x-axis and the semicircle $y = \sqrt{r^2 - x^2}$ is revolved about the x-axis. Find this volume. (Obviously, you must draw a figure and show your work.)
