MA-161 (F,07)
EXTRA CREDIT Problems

Name____________________________

[30 possible]

12. (Unit 5) In the first figure at left is the graph of the function y = f(t). We define a new function, g, by the equation

$$g(x) = \int_{-2}^{x} f(t)\,dt \text{ for } x \text{ in } [-2, 7].$$

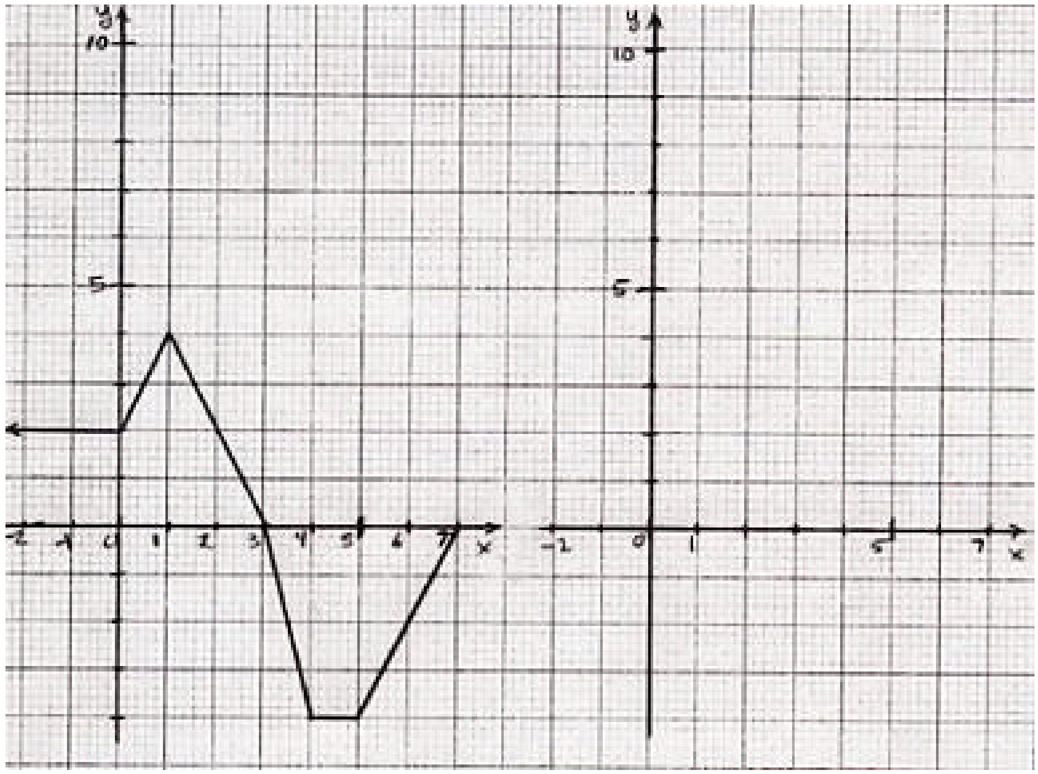
Do the following on your answer sheet:

a. Make a table of values for g where x = –2, –1, ..., 7.

b. On what intervals is g increasing?

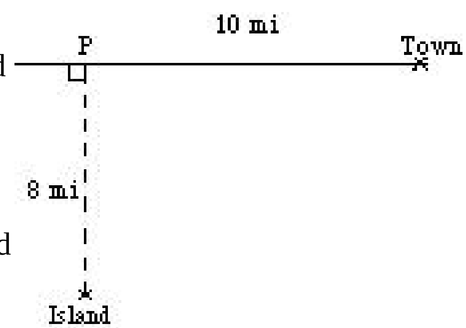
c. Where does g have a maximum value? What is that value?

d. Draw a graph of g on the coordinate system in the second figure.



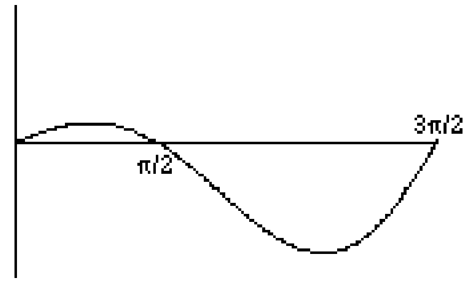
[35 possible]

13. (Unit 3.) A lighthouse is on a small island 8 miles off a straight coast, and a town is located 10 miles down the coast from the point on the coast nearest the lighthouse. Supplies are moved from the town to the lighthouse on a regular basis so it's desirable that they arrive at the lighthouse in the shortest possible time. If supplies can be moved at the rate of 7 miles per hour on water and 25 mph on land, how far from point P should a dock be constructed for the supplies to arrive in a minimum amount of time? [Use calculus to answer this question.]



[30 possible]
14. (Unit 3.) (You may not do this one for EC if you already did it for credit as an alternative question in Problem 4.) An open box is to be made from a 15 inch by 24 inch rectangular sheet of metal by cutting identical squares from each corner and folding up the sides. What are the dimensions and the volume of the largest possible box (that is, the box with largest volume) that can be made from this sheet of metal? (Use calculus.)

[20 possible]
15. (Unit 4.) The graph of $f(x) = x \cdot \cos x$ is given at right. **Use your Riemann sums program to approximate to within two decimal places the values asked for in a and b below.**
Tell which sums and the value of n you're using.



a. The <u>area</u> of the region between the graph of $y = f(x)$ and the x-axis from $p/2$ and $3p/2$.

b. The value of the definite integral of $f(x)$ from $x = 0$ to $x = 3p/2$.

c. Find this derivqative, showing your work. Be sure to simplify your result.
$$\frac{d}{dx}(x \cdot \sin x + \cos x) = x \cdot \cos x.$$

d. Use the Fundamental Theorem of Calculus to find the <u>exact</u> value of $\int_0^{3p/2} x \cdot \cos x \, dx$.

[15 possible]
16. (Unit 4.) Find the following definite integral <u>by substitution</u>. [You <u>must</u> use substitution even if you can find an antiderivative in your head. The thing I'll be looking for here is **the logic of your method**.] Show all your steps logically, using correct notation. The integral to find is $\int_0^4 x\sqrt{25 - x^2}\,dx$.

[20 possible]
17. (Unit 2.) Use implicit differentiation to do the following:

Given that $\frac{d}{dx}(\tan x) = \sec^2 x$, derive the formula for $\frac{d}{dx}(\arctan x)$. Obviously you must show your work. [If you can't do this one, you may, for less credit, obtain the formula for $\frac{d}{dx}(\ln x)$ from the formula $\frac{d}{dx}(e^x) = e^x$.]

[25 possible]
18. (Unit 2.) Differentiate these functions and simplify: (a) $g(x) = (x^2 + 1)^7$ (b) $y = x \cdot \ln x - x$

(c) $h(x) = \dfrac{1 + x}{3 + 4x}$ (d) $y = \sin^3(x^2 + 1)$ (e) $f(t) = A \cdot \sin\left(t\sqrt{\frac{k}{m}}\right)$

[30 possible]
19. (Unit 2.) Here are some useful things that you learned in precalculus:
$$b^x = e^{\ln b^x} \quad \log_b x = \frac{\ln x}{\ln b} \quad \tan x = \frac{\sin x}{\cos x}$$

Use these to <u>derive</u> the formulas for the derivatives of b^x, $\log_b x$, and $\tan x$. To do so, you may use only the derivatives of e^x, $\ln x$, $\sin x$, and $\cos x$ plus things like the Chain Rule, the Product Rule, etc. Obviously you must show your work.

[15 possible]
20. (Unit 4.) Find these integrals. Show your work.

(a) $\displaystyle\int_0^{\ln 2} e^x\,dx$ (b) $\displaystyle\int \left(\sin^3 x\right)\cos x\,dx$ (c) $\displaystyle\int \frac{\tan^{-1}x}{x^2+1}\,dx$

[20 possible]
21. (Unit 4.) Find $\frac{d}{dx}\left(\frac{1}{2}(x + (\sin x)(\cos x))\right)$. Simplify your result, using the Pythagorean identity $\sin^2 x + \cos^2 x = 1$. Then find $\displaystyle\int_0^{p/3} \cos^2 x\,dx$.

[15 possible]

22. (Unit 2.) If $y = e^{x^2}$, find (a) $\frac{d^2y}{dx^2}$ and (b) $y''\big|_{x=0}$.

[20 possible]
23. (Unit 2.) Use <u>implicit differentiation</u> to find $\frac{dy}{dx}$ when y is given implicitly as a function of x by the equation $x\cdot\ln y - x^2 = y^3$.

[20 possible: 5, 5, 10]
24. (Unit 3.) Draw pieces of graphs of $y = f(x)$ near $(1, f(1))$, given the following information about f: [There are <u>three</u> pictures to be drawn. Also, you must include the tangent line in each picture.]

a. $f(1) = 2$, $f'(1) = -1/2$ and $f''(1) < 0$. b. $f(1) = 2$, $f'(1) = 0$ and $f''(1) > 0$.

c. $f(1) = 2$, $f'(1) = 1/2$, $f''(1) = 0$ and $f^{[3]}(1) > 0$.

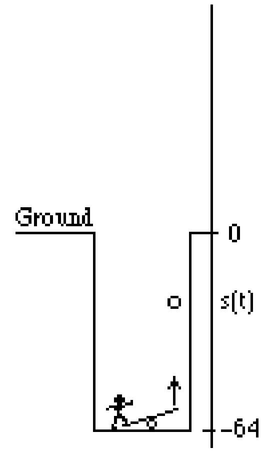
[20 possible]
25. (Unit 2.) To differentiate a function of the form $[f(x)]^{g(x)}$, one uses <u>logarithmic differentiation</u>. Here is a reminder of how this works using $y = x^x$ as an example:

Take the natural log of both sides to get $\ln y = x\cdot\ln x$. Implicitly differentiate both sides with respect to x to get $\frac{1}{y}\cdot y' = x\cdot\frac{1}{x} + \ln x$. Then solve for y' and simplify to get $y' = x^x(1 + \ln x)$.

This method can be used to differentiate functions of the form $[f(x)]^{g(x)}$.

Your problem: Use logarithmic differentiation to find $\frac{d}{dx}\left(x^{\arctan x}\right)$.

[20 possible]
26. (Unit 4.) A ball is catapulted vertically upward from the bottom of a 64 foot hole at the initial velocity of 96 feet per second. $s(t)$ is the position of the ball with respect to the ground t seconds after it is first catapulted. (Up is positive, ground level is zero as shown at right.)



a. We're given that the initial velocity is 96 feet per second, the initial position is 64 feet below ground level. We also know that the acceleration due to gravity is a constant 32 feet per second per second. State these initial conditions in terms of the functions s, s', and s''. Then use antidifferentiation to <u>derive</u> an equation for $s(t)$. (Obviously you have to show your work.)

b. When does the ball reach its apex? How high is it then?

c. When is the ball at ground level? What is the velocity of the ball then?

[20 possible]
27. (Units 2 & 4.) One of the formulas in the Table of Integrals (which you will learn to use in Calculus II) is $\int \sqrt{a^2 - u^2}\,du = \frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\arcsin\left(\frac{u}{a}\right) + C$ where a is a positive constant. Show that this formula is correct .

[15 possible]
28. (Unit 5.) Use the formula in Problem 27 to show that the area of a circle with radius r is πr^2.